

Grade Modelling with Local Anisotropy Angles: A Practical Point of View

David F. Machuca-Mory^{*1}, Harri Rees², and Oy Leuangthong³

¹SRK Consulting, Toronto, Canada, Senior Consultant (Geostatistics), +1-416-848-3214,

dmachuca@srk.com

²SRK Consulting, Toronto, Canada, Consultant (Geology), +1-416-601-1445, hrees@srk.com

³SRK Consulting, Toronto, Canada, Principal Consultant (Geostatistics), +1-416-848-3220,

oleuangthong@srk.com

ABSTRACT

Resource geologists often find geostatistical algorithms that rely on a single stationary model of spatial continuity inadequate for modelling grades in structurally complex deposits. Current practical methods for dealing with this challenge can be grouped in either coordinate transformations or locally changing spatial continuity parameters. A common feature of both groups is the requirement of a prior model of local anisotropies for the spatial continuity of grades. This model would include local angles of spatial continuity, and for some methods, also the local anisotropy ranges of continuity and other parameters. The local angles model can be built from direct measurements of dips and orientations of the geological structures that control the spatial distribution of grades. However, these measurements are seldom as exhaustive as needed. Thus, the local angles model is often completed by the information provided by geological interpretations in the form of wireframes. The local anisotropy ranges model is more difficult to construct. So, the local anisotropy ratios are often deemed constant and equal to an anisotropy ratio derived from the stationary variogram model. This paper focuses on various practical aspects of the construction of a model of local anisotropy angles from geological wireframes. The topics presented include the methods for obtaining the local anisotropy angles, the motivation and methods to pre-process these angles, and the approaches available to construct a model of local anisotropy angles field for use in mineral resource estimation. Statistics based on the inner product of vectors are presented as simple tools for comparing alternative fields of local anisotropy angles. Kriging with local anisotropy angles is applied to a structurally-controlled gold deposit in Ghana, and the impact of using pre-processed angles and alternative angle fields is assessed. Additionally, grade estimation with local anisotropy angles is benchmarked against estimation with ordinary kriging with stationary variograms.

*Corresponding author: SRK Consulting Toronto, Senior Consultant (Geostatistics), Suite 1300-151 Yonge Street Toronto, ON, Canada, M5C 2W7. Phone: +1 416 8483214. Email: dmachuca@srk.com

1. Introduction

Most mineral deposits are characterized by complex geology accompanied by complex geometry. In these instances, the use of a single variogram model that captures the general orientation and continuity structure is unable to capture local changes in geometry. To circumvent this issue, the three-dimensional model of the deposit can be divided into domains where a single stationary variogram model can be used as a reasonable representation of the continuity of the enclosed grades. Often times, however, the actual domains are deformed by folding, shearing, and other structural processes (see Figure 1a).

Several approaches can be taken to deal with folded domains under a conventional geostatistical framework. One idea is to sub-domain on the basis of structural orientation, wherein stationary variograms are calculated and modelled. This is not always practical if the domain's geometry is so complex as to warrant a large number of structural sub-domains. A second approach is to unfold the deformation. However, it can be difficult to unfold the structure when deformation happens along more than one axis or in the presence of reverse faults (Deutsch and Rossi, 2013). Finally, a variety of methods have emerged in the last two decades associated with locally defined anisotropy.

This last group of local anisotropy approaches differs in how they obtain and use the local anisotropies field. An early approach to anisotropy field generation proposed by Haas (1990) relies on an automated fitting of a variogram calculated with data within a moving window. A related approach, called Moving-GeoStatistics, (Magneron et al., 2010) obtains local anisotropy angles and ranges by optimizing the kriging cross-validation within a neighbourhood. These methods, where the local anisotropies are implicitly obtained during the grade estimation process, are different from the methods that rely on an explicit local anisotropy field. These include the dynamic anisotropy (DA) method (CAE Mining), local anisotropy kriging (LAK) (Te Stroet, and Snepvangers, 2005; Isaaks, 2005), distance weighted local variograms (DWLVs, Machuca-Mory and Deutsch, 2013), and locally varying anisotropy (LVA) method (Boisvert et al., 2009).

The DA method changes the variogram orientation and sample search ellipsoid according to the local anisotropy angles. We note that the DA approach is not new, and is available on some commercial platforms such as Datamine Studio 3. The LAK uses the local anisotropies to improve the variogram inference and also to adjust the search neighbourhood. The DWLVs can be used to model the LVA field if data is abundant enough to produce robust local variograms. If data is scarce, the structural geology knowledge of the deposit can inform the modelling of local variograms. Figure 1B shows the basic idea behind the locally changing anisotropic search ellipsoids in DA, LAK, and DWLV. The LVA method uses the field of local anisotropies to infer the variogram and model the grades using non-Euclidean distances. It is beyond the scope of this paper to present a detailed discussion of these four methods and their comparative advantages and disadvantages.

This paper focuses on the construction of the exhaustive local anisotropy field required by all of these methods. We present a review of the practical aspects of the construction of the local anisotropy field and an assessment of the impact of angles pre-processing in the grade modelling. For illustrative purposes, the steps for building the local anisotropy field and its use in grade modelling will be demonstrated using a small area of a folded gold deposit in Ghana.

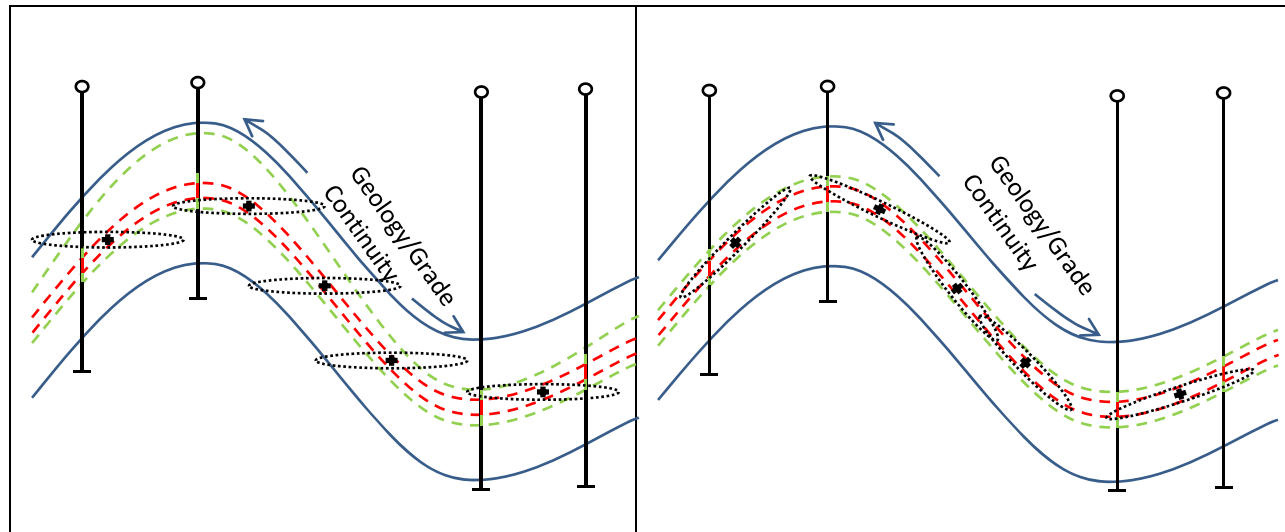


Figure 1: Schematic illustration of anisotropic ellipsoids in presence of a single stationary variogram (left) and using locally varying anisotropies (right).

2. Building the Local Anisotropy Field

The process to constructing a local anisotropy field generally consists of three steps: (1) obtaining the initial anisotropy orientation angles, (2) filtering or cleaning of these initial angles, and (3) interpolation of these angles at the same resolution of the final grade block model for the purposes of grade estimation. This last step is necessary since the local angles that come from the wireframe or strings seldom cover all locations of the block model or may contain more than one dip or dip direction values per block. Each of these steps is discussed below.

Getting the Local Angles

Geological objects are often represented as wireframe surfaces or solids. The construction of these wireframes involves collecting all the information relevant to the mineralization from borehole logs, structural geology interpretations, and other analogue information, and interpreting this information into a coherent representation of the underlying geology. A key aspect to the mineral resource evaluation of a deposit is that the geology model should define the most important mineralization controls (Deutsch and Rossi, 2014).

When strong structural controls are present, it should be possible to extract information about the local grade continuity from geological wireframes or strings. The former approach is based on extracting dip and dip directions using the triangulated facets of a geological solid to control orientation during mineral resource evaluation. This is available in commercial software and is often the first step towards the construction of the local anisotropy field. However, these wireframes may show details at a smaller resolution than the data spacing or they may even contain noise and discontinuities that are not related to the structural geology but to artefacts introduced during the wireframing process. In these cases, the local dip and dip-direction angles have to be pre-processed before they can be used to build a suitable local anisotropy field.

The latter and alternative approach to using the wireframe surfaces or solids is to digitize strings within the domain to conform to grade continuity and/or to follow the geological structures. A significant benefit to this approach is the flexibility to define local angles of grade continuity that diverge from the structure's dip and dip directions (see Figure 2). The price of flexibility in this instance is professional time and effort. However, the approach should yield smoother anisotropy fields and require less cleaning of the resultant angles since the modeller may eliminate abrupt changes while digitizing the strings.

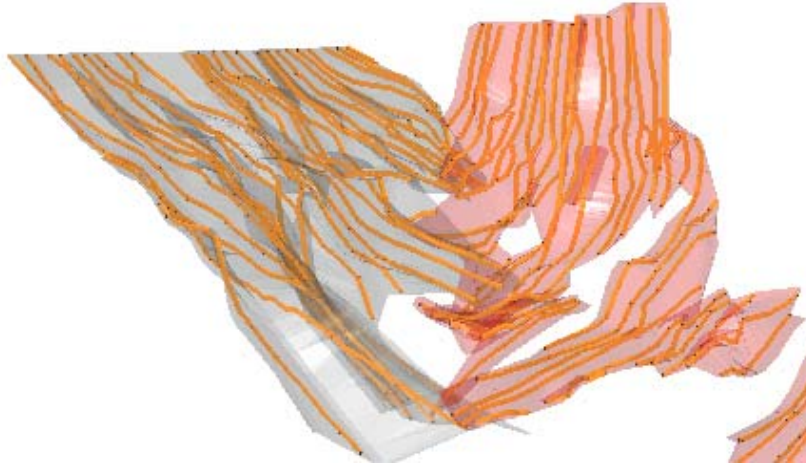


Figure 2: Digitized strings for anisotropy angles.

Smoothing the Local Angles

Local dip and dip-direction angles can be extracted from the individual facets of these wireframes. This may introduce facets that result in spurious dip and dip angles. For example, Figure 2 (left) shows the tops and the sides of a solid that resulted in angles that diverge abruptly from the shape of the modelled structure. Also, local angles obtained from wireframes that present excess detail or abrupt changes (see Figure 3, right) may translate into local search ellipsoids that would have difficulty gathering sets of neighbouring samples that are consistent with adjacent locations. These issues are strong motivations for pre-processing the local angles obtained from wireframes.

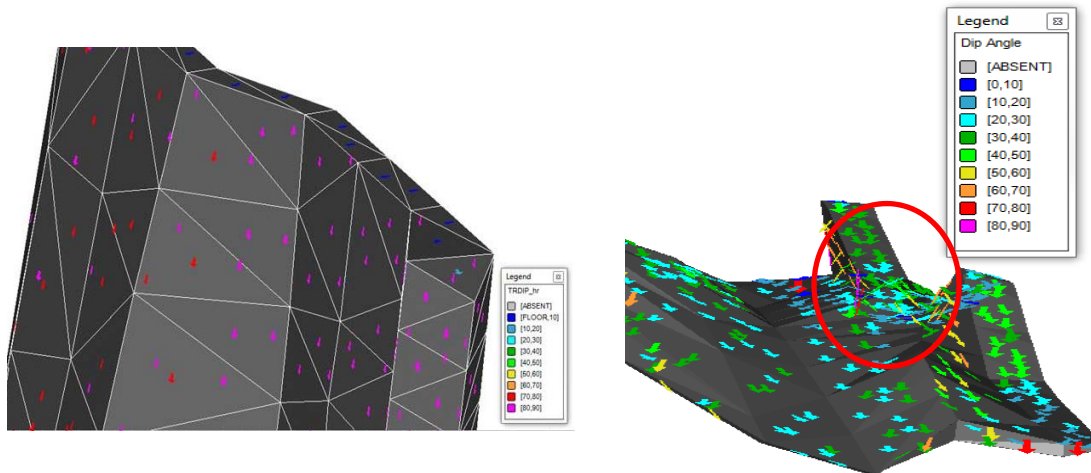


Figure 3: Problematic dip and dip directions obtained from wireframes (tops and ends [left], abrupt changes [right]).

A first step in the pre-processing of angles is the manual cleaning of the wireframe edges (see Figure 2). Not all noise can be efficiently removed by the manual cleaning of angles. Thus, a second pre-processing step may involve the smoothing of angles. The goal of this is to produce a set of angles that describe the local trends in grade continuity at the modelling scale without blurring relevant local features.

We propose an approach to smoothing that is performed on the vector components of the unit vectors formed by the dip and dip directions obtained from facets or strings. The x, y, z components of the unit vector are obtained from

$$\begin{aligned}
x &= \sin(\varphi) * \cos(\theta) \\
y &= \cos(\varphi) * \cos(\theta) \\
z &= \sin(\theta),
\end{aligned}
\tag{1}$$

where θ is the dip (negative downwards) and φ the dip direction, both expressed in radians. The smoothed $\bar{\theta}$ and $\bar{\varphi}$ angles can be recovered from the smoothed vector components $\bar{x}, \bar{y}, \bar{z}$ by

$$\begin{aligned}
\bar{\varphi} &= \text{atan2}(\bar{x}, \bar{y}) \\
\bar{\theta} &= \arcsin(\bar{z}).
\end{aligned}
\tag{2}$$

Different smoothing techniques can be applied, but a moving average filter is straightforward and sufficient for this purpose. The smoothing technique used is not as important as the degree of angle smoothing. On one hand, we need to trim or minimize angles that are outliers or that change abruptly, and on the other, it is important to preserve the trends that control the grade continuity and to avoid introducing biases in it. Striking the balance between these two objectives is key.

Comparative global statistics of the original versus smoothed angles can be used to assess the degree of smoothing. The range of smoothed angles is compared with the range of cleaned angles to check if the variability warranted by the structural geology is preserved. The average dip and dip direction must be also preserved, otherwise it would mean that biases were introduced during smoothing. This, however, does not ensure that relevant local folds and/or features are adequately retained.

A practical local check to compare individual original and smoothed angles is to calculate and analyze the dot product of their vector components. If $\mathbf{D} = (x, y, z)$ is the vector defined by the original dip, θ , and dip direction, φ , angles and $\bar{\mathbf{D}} = (\bar{x}, \bar{y}, \bar{z})$ are defined by the smoothed angles $\bar{\theta}$, and dip direction, $\bar{\varphi}$, the dot product is expressed as

$$\mathbf{D} \cdot \bar{\mathbf{D}} = x \cdot \bar{x} + y \cdot \bar{y} + z \cdot \bar{z} = \cos \alpha_{\mathbf{D}, \bar{\mathbf{D}}},
\tag{3}$$

With $\alpha_{\mathbf{D}, \bar{\mathbf{D}}}$ as the angle between the vectors defined by the original and smoothed dip and dip directions (see Figure 4). If these angles did not change, the dot product is 1, if the smoothed angles define a perpendicular direction to the original, the dot product is zero, and it is negative if the smoothed direction is opposite to the original. Therefore, pre-processing of the local angles should result in a high proportion of dot product values that are 1 or close to 1 dot and a low number of null dot product values. Negative dot product values that are -1 or close to -1 are also acceptable, since variograms and search ellipsoids are symmetric. Visualizing the dot product values is useful to identify locations where angle smoothing resulted in significant changes from the original angles.

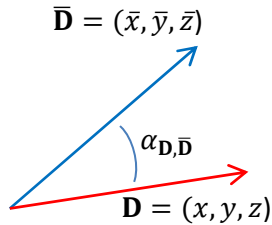


Figure 4: Raw, \mathbf{D} , and smoothed, $\bar{\mathbf{D}}$, vectors and the angle α between them.

Interpolating the Local Angles

The estimation algorithms (DA, LAK, DWLVs, and LVA) that work with local anisotropies require exhaustive fields of local anisotropy angles. Thus, the smoothed local angles have to be interpolated since they may not populate all the locations to be estimated or they may contain information that may be considered as collocated at the grade modelling resolution.

The interpolation of local angles is performed on the vector components as in Equation (1). Subsequently, they are recombined using Equation (2) at every location of the model to form the final anisotropy field. Again, the interpolation method is not as important as the smoothing introduced by it. The range of local angles should be preserved, as well as the average dip and dip directions. If local angles are obtained using a wireframe approach, then search radii for estimating angles should be highly localized, whereas a string approach to angle generation may require longer search radii and is dependent on the number of strings available.

3. Application and Impact of Local Anisotropy Angles

The Wassa gold mine is located approximately 40 kilometres northeast of the town of Tarkwa, Ghana. The mine lies within the southern Ashanti greenstone belt, and the gold mineralization is associated with quartz veining hosted in polydeformed greenstone rocks. The gold mineralization is structurally controlled, related to vein densities and sulphide content.

The deposit has been affected by several generations of deformation. The most significant resulted in the folding of the gold-bearing strata. Only two of the many veins that comprise the gold-bearing strata were selected for this case study. Mine geologists interpreted these strata in sections and used them to create two wireframe models, one on each side of the fold axis. The dip and dip directions of each wireframe facet were obtained. Figure 5 (left) shows the dips angles superimposed to the wireframes, from which they were extracted. A first cleaning of the anisotropy angles was performed by removing those angles that are artefacts of the wireframe construction, such as tops and edges of the solids. A scatterplot of dip and dip directions colored by elevation can be useful for identifying and removing these distracting angles (see Figure 5 right).

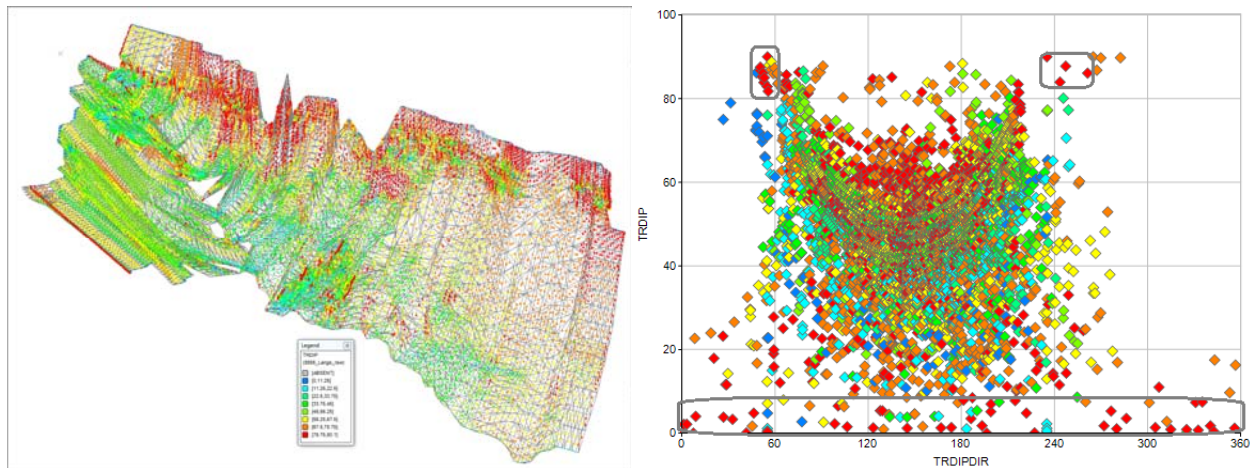


Figure 5: Wireframes of the estimation domains and the local dip angles obtained for each of their facets (left), and dip and dip direction scatterplot colored by the elevation of facet from where the angles were taken (right). The areas encircled correspond to facets at the top and at the end of the wireframes.

In order to remove locally irregular angles more efficiently than a manual approach, moving average filters of different window sizes were applied to the x , y , z components of the vectors formed by dips and dip directions (see Equation 1). The degree of smoothing imposed by these filters was assessed by the dot product (Equation 3) of original vs. smoothed angles. Figure 6 shows the dot product values for the moving windows of size 20m x 20m x 10m (left) and 80m x 80m x 40m (right). Areas with small or negative dot product values are those where smoothing has a greater impact. Statistics of the dot product values can also be used to assess the impact of smoothing these angles. For instance, if the dot product mean is close to 1 and the standard deviation is small, it would mean the number of smoothed angles that diverge from the original angles is small. A mean value smaller than 1 and a large standard deviation may

indicate that angles were overly smoothed. Also, the arccosine of the dot product values can be used to quantify the proportion of smoothed angles that diverge from the original angles by more than an angular threshold that is deemed acceptable. Figure 7 shows the standard deviation of dot product values and the percentage of smoothed angles that differ by less than 15° from the original angles for different moving window sizes. In that graph, the standard deviation of dot products rise quickly as the moving window increases up to a size of 40m x 20m x 10m, where it starts to stabilize. For a moving window of 20m x 20m x 10m the proportion of angles modified by more than 15° is already around 50% of the total and it increases steadily with the search neighborhood size. In view of these statistics and the location of the most impacted angles, it was decided that a window of 20m x 20m x 10m was big enough to remove local irregularities without compromising the spatial structure informed by the structural geology.

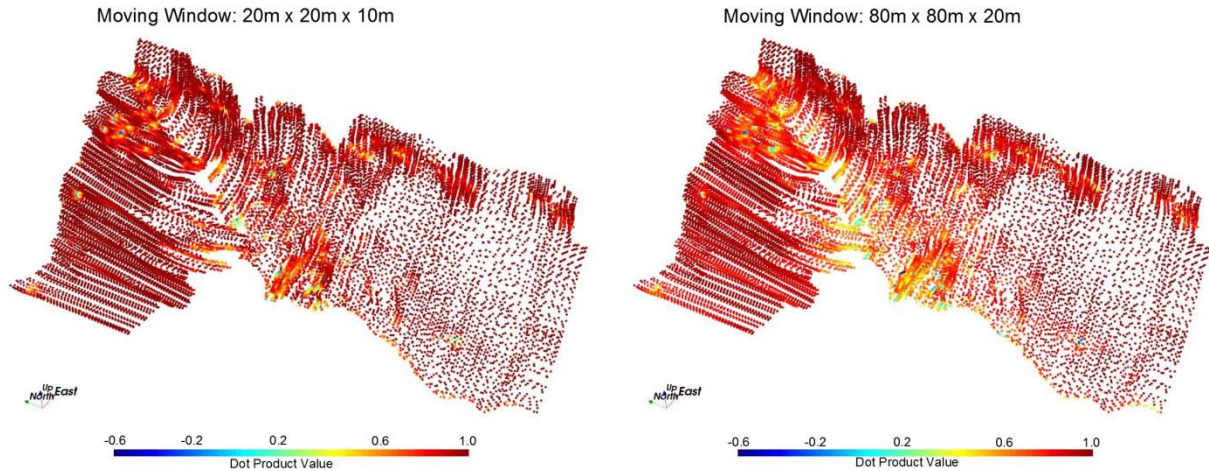


Figure 6: Dot product values calculated between original angles and smoothed angles using a moving average window of 20m x 20m x 10m (left) and 80m x 80m x 20m (right).

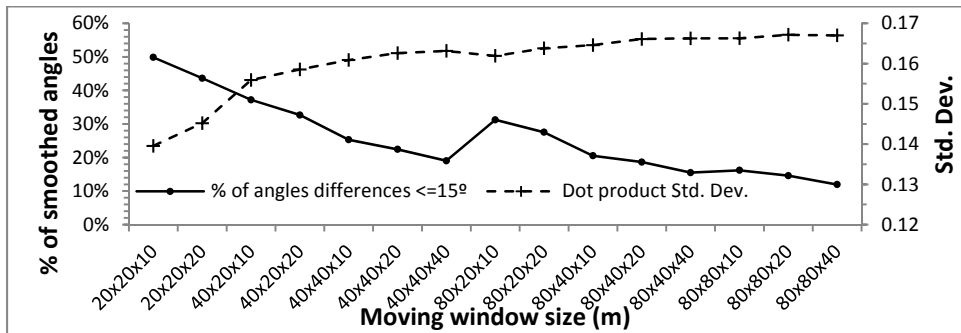


Figure 7: Standard deviation of the dot product of raw versus smoothed local angles and the proportion of smoothed angles that differ 15 degrees or less from the original angles.

Building an exhaustive local anisotropy field by interpolating irregularly spaced local angle values may require the use of a large search window. This introduces another degree of angle smoothing that has to be controlled. The idea is thus to use the smallest search window in order to populate all, or most of, the estimation domain. For this case study, the final local anisotropy field was built using inverse distance weighting estimation of the smoothed angles with a search window of 30m x 30m x 20m. Figure 8 shows views of the dip and dip direction fields.

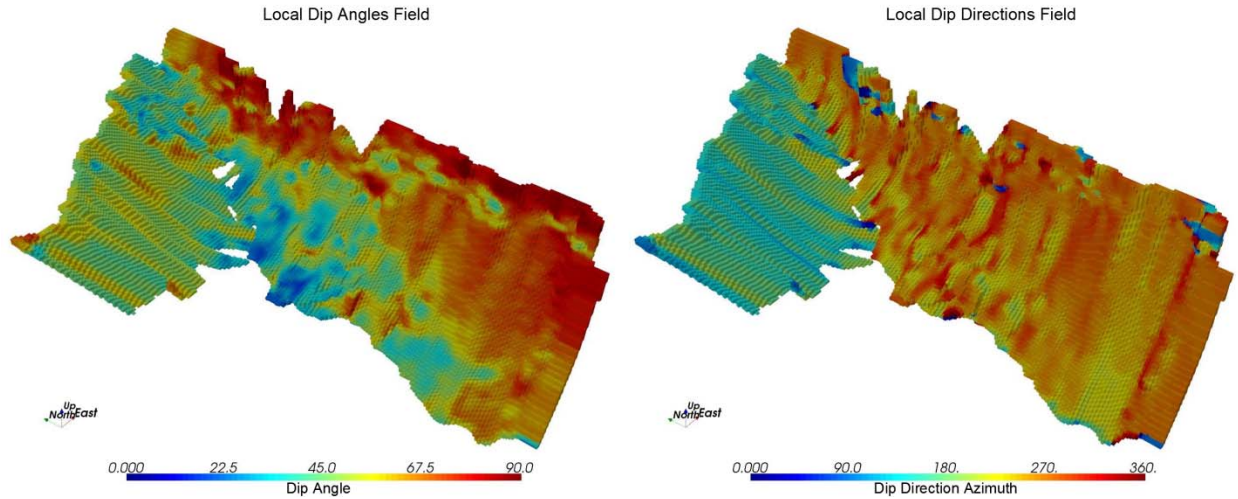


Figure 8: Local dip angles (left) and local dip directions (right) exhaustive fields.

Three estimated models of gold grade were generated: (1) using two different global anisotropies were used for each side of the fold; (2) using a local anisotropy field built by interpolating the original angles; and (3) using a local anisotropy field based on the smoothed and interpolated angles shown in Figure 8. Table 1 presents the parameters of the variogram model, as well as the orientations of the stationary anisotropies at each side of the fold for Model 1.

Table 1: Stationary variogram model parameters for each side of the fold.

Side	GSLib Rotations				Structure 1					Structure 2				
	Ang 1	Ang 2	Ang 3	N. E.	Sill	Type	R. 1 (m)	R. 2 (m)	R. 3 (m)	Sill	Type	R. 1 (m)	R. 2 (m)	R. 3 (m)
East	260	-55	0	0.2	0.5	Exp.	20	20	7	0.3	Sph.	125	150	12
West	160	-45	0	0.2	0.5	Exp.	20	20	7	0.3	Sph.	125	150	12

Level plans (Figure 9) of the resultant block models show that the use of local anisotropies has a greater impact on the estimated grades than the smoothing of angles. This is as it should be. The smoothing step mitigates the impact of any anomalous angles and should not distract from the key local features that are targeted for model reproduction. In fact, the impact of smoothing should be most noticeable in heavily contorted regions characterized by very tight folds.

Crossplots (Figure 10) of estimated grades between the various models show greater differences in estimated gold grades between the models built using local anisotropies and the model using a stationary global anisotropy. Differences between the two local anisotropy models are minor, as expected. The most significant differences across all three models occur in regions where the local anisotropies depart considerably from the global anisotropy (Figure 11). Differences in the estimated grades between the global and local (original) anisotropy model can be as high as ± 10 g/t gold in this case, which is roughly equivalent to 6.5 times the standard deviation of the estimated values (Figure 11 left). A similar comparison of the models built using different local anisotropy fields (Figure 11 right) shows that while the largest differences tend to appear in the same regions as when comparing against Model 1, these appear now less abundant and more scattered.

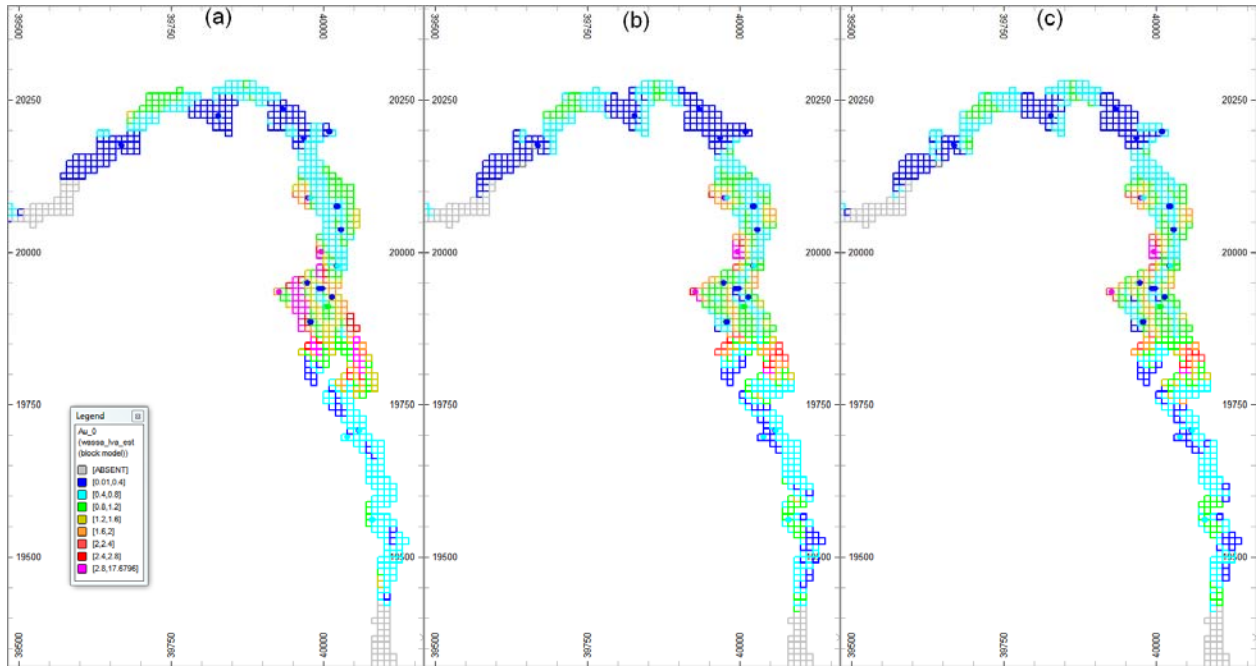


Figure 9: Horizontal sections of the gold grade models built using (a) a single variogram orientation at each side of the fold (Model 1), (b) a local anisotropy field obtained from raw interpolated angles (Model 2), and (c) a local anisotropy field obtained from smoothed and interpolated angles (Model 3).

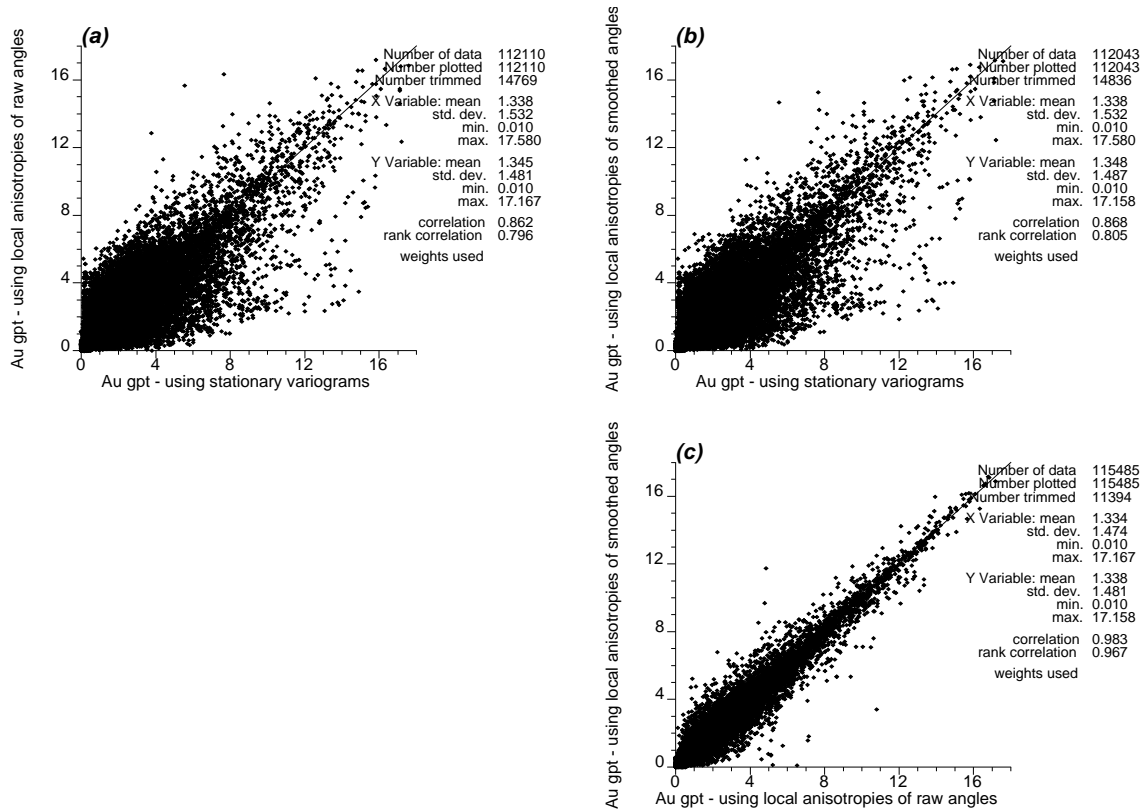


Figure 10: Scatterplot of gold estimates obtained using stationary variograms vs. using a local anisotropy field of raw angles (a), using stationary variograms vs. using a local anisotropy field of smoothed angles (c), and using local anisotropy fields of raw vs. smoothed angles.

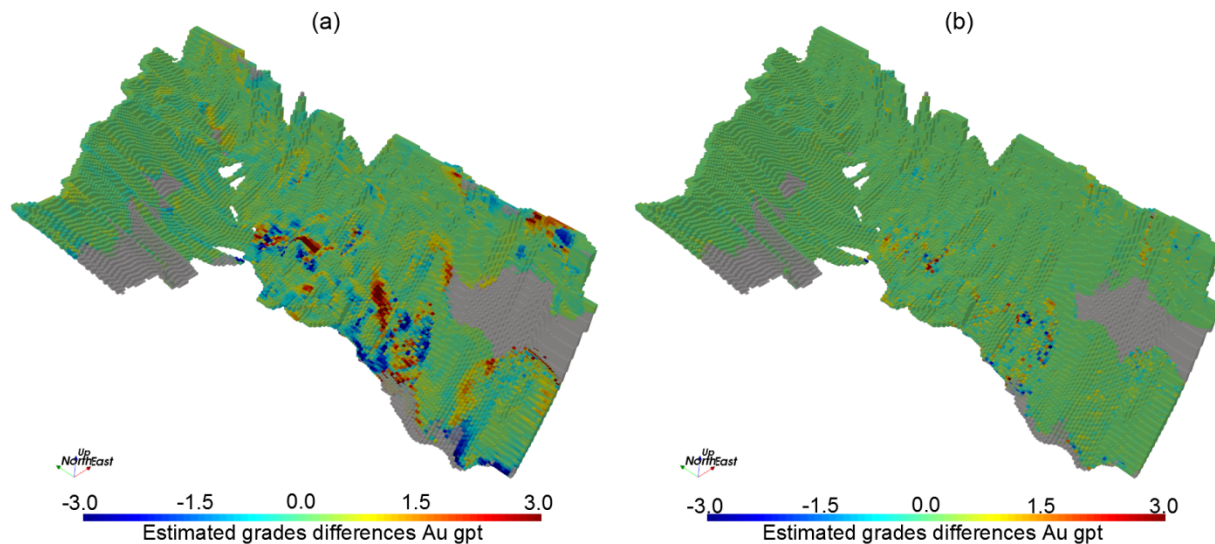


Figure 11: Differences of estimated gold grades in (a) Model 1 minus Model 3, and in (b) Model 2 minus Model 3. Grey areas are un-estimated blocks.

4. Discussion and Conclusions

The use of local anisotropies can have a large local impact in kriged grades. This impact is related to the definition of the local anisotropy field. Thus, the construction of this field is a step that requires special attention when modelling the mineral resources of heavily deformed ore deposits. When this field is built from geological wireframe solids, it is often necessary to pre-process the local dip and dip direction data. This is accomplished by removing spurious angles and applying a certain degree of smoothing in order to impose directions of grade continuity that are more related to the structural geology rather than to artifacts of wireframe generation.

As for any data smoothing exercise, there is a trade-off between light smoothing for filtering out noise and a heavy smoothing that blurs data structure. The dot product of the raw and smoothed vectors defined by the local dips and dip directions is a simple, yet effective tool to simultaneously measure the changes in local angles imposed by smoothing. The statistics of these dot product values can be used to quantify the model-wide impact of local angles pre-processing. Depending on the complexity of the geological wireframes, the practitioner may decide the degree of local angles pre-processing appropriate to their deposit. In some cases, the smoothing of angles may not be needed, since the subsequent interpolation required to build the exhaustive local anisotropies field may filter out those spurious angles that could not be deleted manually.

An important aspect beyond the scope of this paper is the impact of kriging with local anisotropy fields on resource classification schemes. Local anisotropies modify the number and weighting of neighbouring data used for estimation at each location. This is relevant for classification schemes based on number of conditioning samples, number of boreholes, average distances to the data, and kriging variances. This subject remains for further investigation.

Acknowledgments

The authors would like to thank Golden Star Resources Limited for providing data from its Wassa deposit for this study.

References

- Boisvert, J. B., Manchuk, J. G., & Deutsch, C. V. 2009. Kriging in the presence of locally varying anisotropy using non-Euclidean distances. *Mathematical Geosciences*, 41(5): 585-601.
- CAE Datamine Corporate Limited. Dynamic Anisotropy. 2014. CAE Mining Documentation for Studio 3, Version 3.21. Accessed on November 2014.
- Haas, T. C. 1990. Kriging and automated variogram modeling within a moving window. *Atmospheric Environment. Part A. General Topics*, 24(7): 1759-1769.
- Isaaks, E. 2005. Local Anisotropy Kriging. Downloaded from www.isaaks.com, September 2014.
- Machuca-Mory, D. F., & Deutsch, C. V. 2013. Non-stationary geostatistical modeling based on distance weighted statistics and distributions. *Mathematical Geosciences*, 45(1): 31-48.
- Magneron, C., Jeanne, N., Le Moine, O., and Bourillet, J. F. 2010. Integrating prior knowledge and locally varying parameters with moving-geostatistics: Methodology and application to bathymetric mapping. In *geoENV VII—Geostatistics for Environmental Applications*. Springer Netherlands. pp. 405-415).
- Rossi, M. E., & Deutsch, C. V. 2013. *Mineral Resource Estimation*. Springer Science & Business Media. P. 44.
- Te Stroet, C. B., and Snepvangers, J. J. 2005. Mapping curvilinear structures with local anisotropy kriging. *Mathematical geology*, 37(6): 635-649.