Radial basis functions and kriging – a gold case study

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Introduction

Recent advances in mining software have enabled the implementation of radial basis functions (RBFs) as interpolation and extrapolation algorithms for both continuous (grade) and categorical (geology) data.

RBFs approximate a specific type of kriging called Dual Kriging (Horowitz *et al* 1996). A recent paper by Stewart *et al* (2014) details some of the theory of RBFs and kriging and compares estimates using a simulated data set from a negatively skewed distribution (approximating low grade iron ore). RBFs are the underlying algorithms used in the Leapfrog software which is used for this case study.

This article presents a less theoretical and more empirical look at RBF estimates using a real gold data set for which both exploration drilling and grade control drilling data sets are available. This gold distribution is a positively skewed distribution which typically presents more challenges during estimation due to the sensitivity of estimate to the high grade tail of the distribution in comparison to normal or negatively skewed distributions.

The presentation is largely pictorial as a more detailed paper is in planning for presentation at an upcoming conference.

Data set

The data is from an area of the Goodall open cut gold mine in the Northern Territory know as A-Pod. Mining of A-Pod was completed in 1992 and all pre-mining exploration data and production grade control assays are available. A brief description of the geology is given below. A full description can be found in Quick (1991).

The mineralisation occurs on the eastern limb of an anticline in a well-defined sub-vertical zone that measures up to 50 m in width, 800 m along strike and up to 140 m in depth. The folding is related to the F1 Howley anticline and has formed an open upright anticlinal fold slightly overturned to the west. Dykes have intruded the sequence after the main folding and cross-cut the fold axis.

The gold mineralisation is epigenetic, structurally controlled and is associated with thin (5-50 mm) arrays of quartz-sulfide veins, which bulk to around 5-20 per cent of the rock. The mineralisation occurs primarily within the sulfides. Grades are slightly higher along the eastern margin and lower in the centre of the mineralised zone.

That data consists of an exploration database and a grade control database. With the exception of a missing grade control dataset for one bench, both exploration and grade control data cover the area that was mined by open pitting.

For analysis purposes we will consider subsets of the exploration composites that coincide with the estimated blocks that contain six or more grade control composites. The ordinary kriged block estimate from the grade control composites within the modelled volume will be considered as our definitive data set (reality).

The resource is drilled on 50 m spacing along strike to a depth of approximately 150 m. Infill drilling is spaced at 25 m along strike to a depth of 50 m on average. The average sample length is 1 m.

There are numerous grade control patterns throughout the pit, including 2.0 × 5.0 m, 2.0 × 4.5 m, 3.0 × 4.0 m, 3.0 × 8.0 m. Most are sampled at 1.5 m intervals over a 6 m bench but holes can vary from 1.5-9 m in total depth. Figures 1 and 2 show the exploration and grade control data sets respectively.

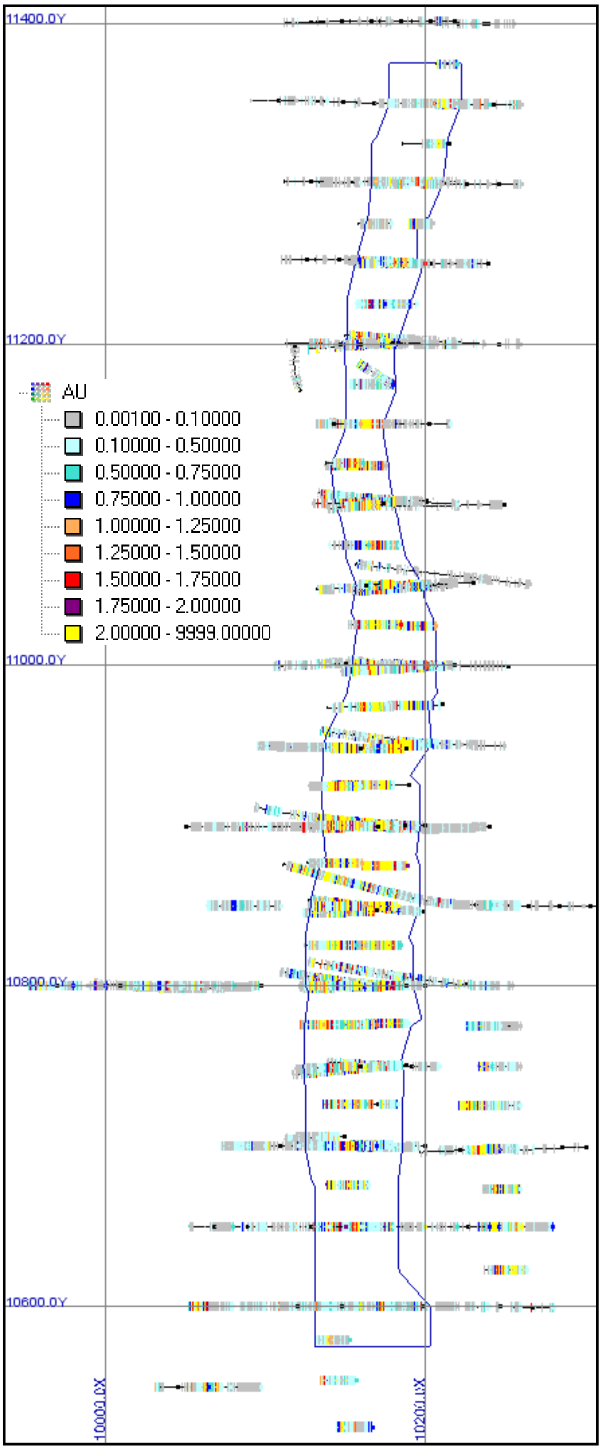

Figure 1: Exploration data set plan view

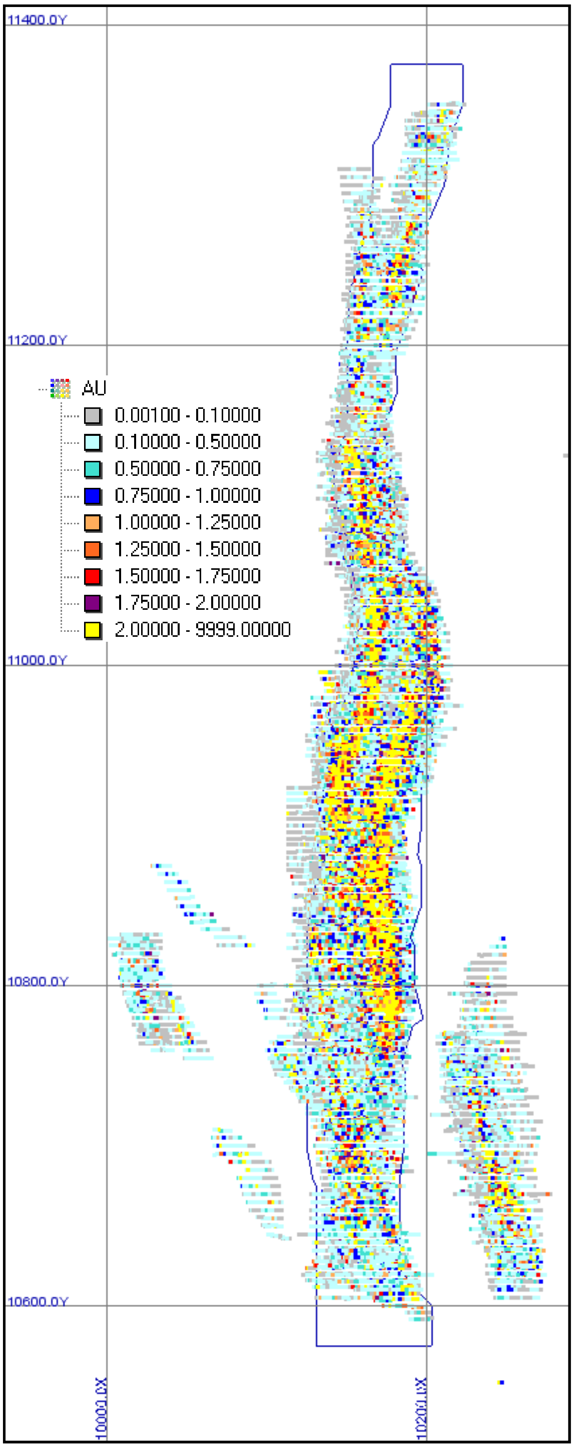

Figure 2: Grade control data set plan view

Gold distributions are typically skewed and can be difficult to estimate, particularly where the high end of the grade and tonnage curves are concerned. Figure 3 shows the histogram of composited Au values used for estimation. The maximum value is 30.4 ppm. No top cutting was used in any of the estimation processes.

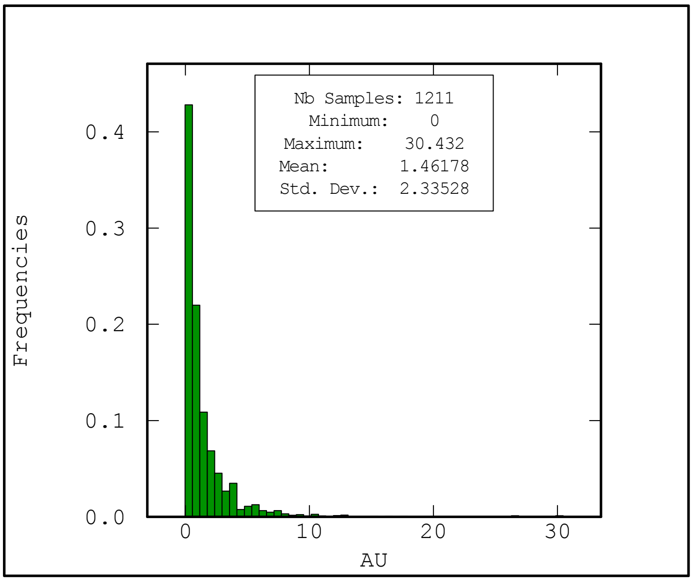

Figure 3: Histogram of composited samples used for estimation

Estimation models

Five models are compared in this article. Each one is described in (Table 1). The models are all estimated on 12 × 25 × 6 m (X, Y, Z) blocks. This block size approximates the exploration drill spacing.

Table 1: The compared models

Ordinary Kriging (OK) 80	OK from exploration date into 12 × 25 × 6 m blocks with a search neighbourhood optimised to give the best average kriging regression slope. This neighbourhood uses a maximum of 80 samples. Variography was modelled from the exploration data.
Simulation1 (Sim1)	A single sequential Gaussian conditional simulation from exploration data that was simulated on a 3 × 6 × 3 m point grid and regularised to 12 × 25 × 6 m blocks. Variography was modelled from the exploration data and is the same as that used for OK.
RBF Regularised (RBF Reg)	RBF interpolant based on exploration data estimated onto a 3 × 6 × 3 m point grid and regularised to 12 × 25 × 6 m blocks. Variography was set to *spheroidal* and the nugget and range set to approximate the variography used for OK. Exact replication of the variogram model was not possible due to the limitations of the RBF interface. The drift parameter was set to *constant* so as to approximate OK. The Transformation parameter was set to *none* so that the process worked directly with the skewed Au grade distribution.
RBF Direct	RBF interpolant based on exploration data estimated directly onto a 12 × 25 × 6 m point grid. Parameters were the same as for RFB reg.
Grade Control (Reality)	OK from grade control data into 12 × 25 × 6 m blocks with a search neighbourhood optimised to give the best average kriging regression slope. Variography was modelled from the grade control data.

Results

The results are compared both globally by grade and tonnage curves and locally by scatterplots of the individual block grades. The results are shown in Figures 4 - 8. The legend on the right hand side of the figures refers to the number of pairs of blocks in each cell of the scatterplot.

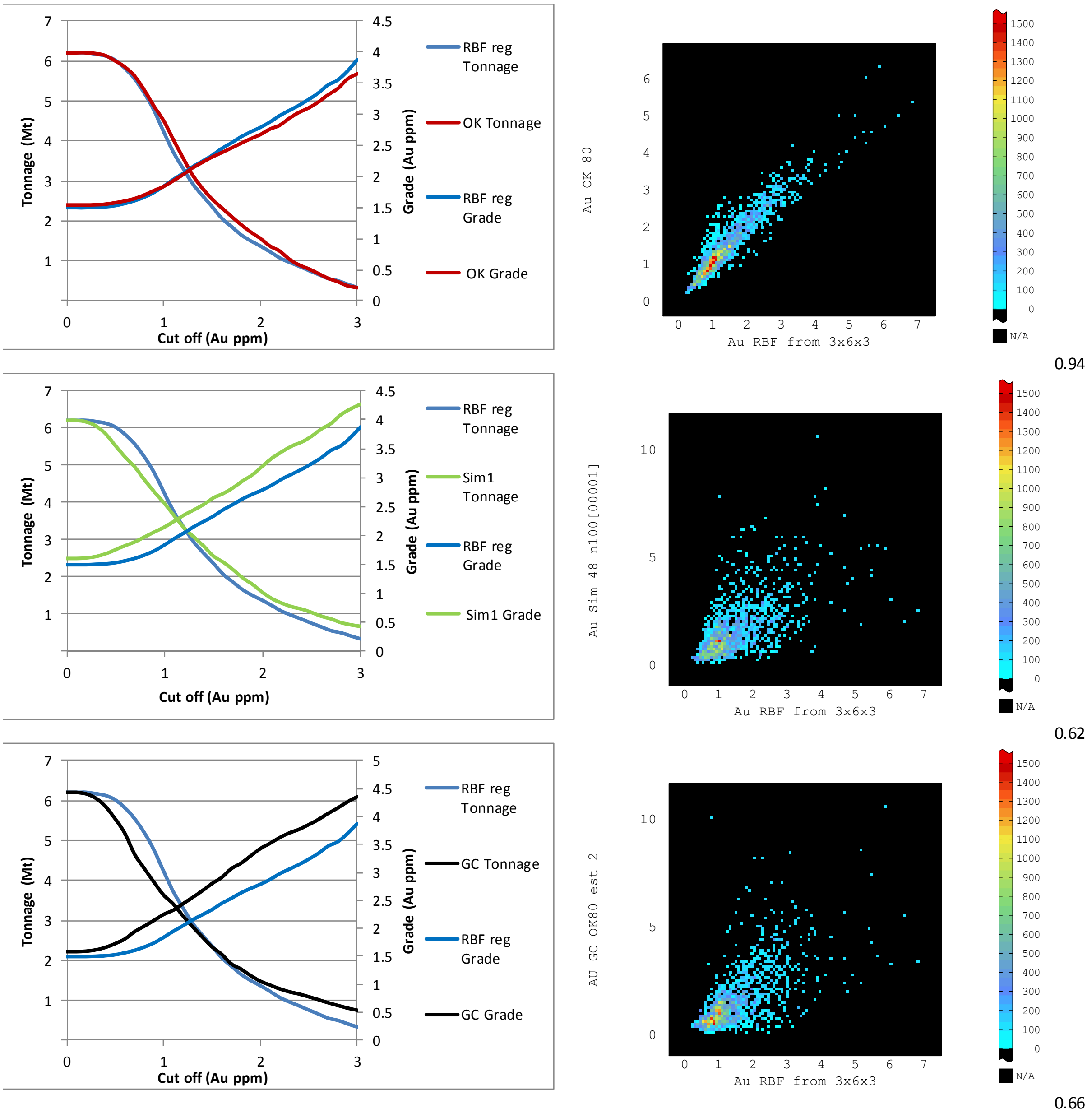

Figure 4: Radial Basis Functions Regularised compared with Ordinary Kriging 80, Sim1 and Grade Control (Reality)

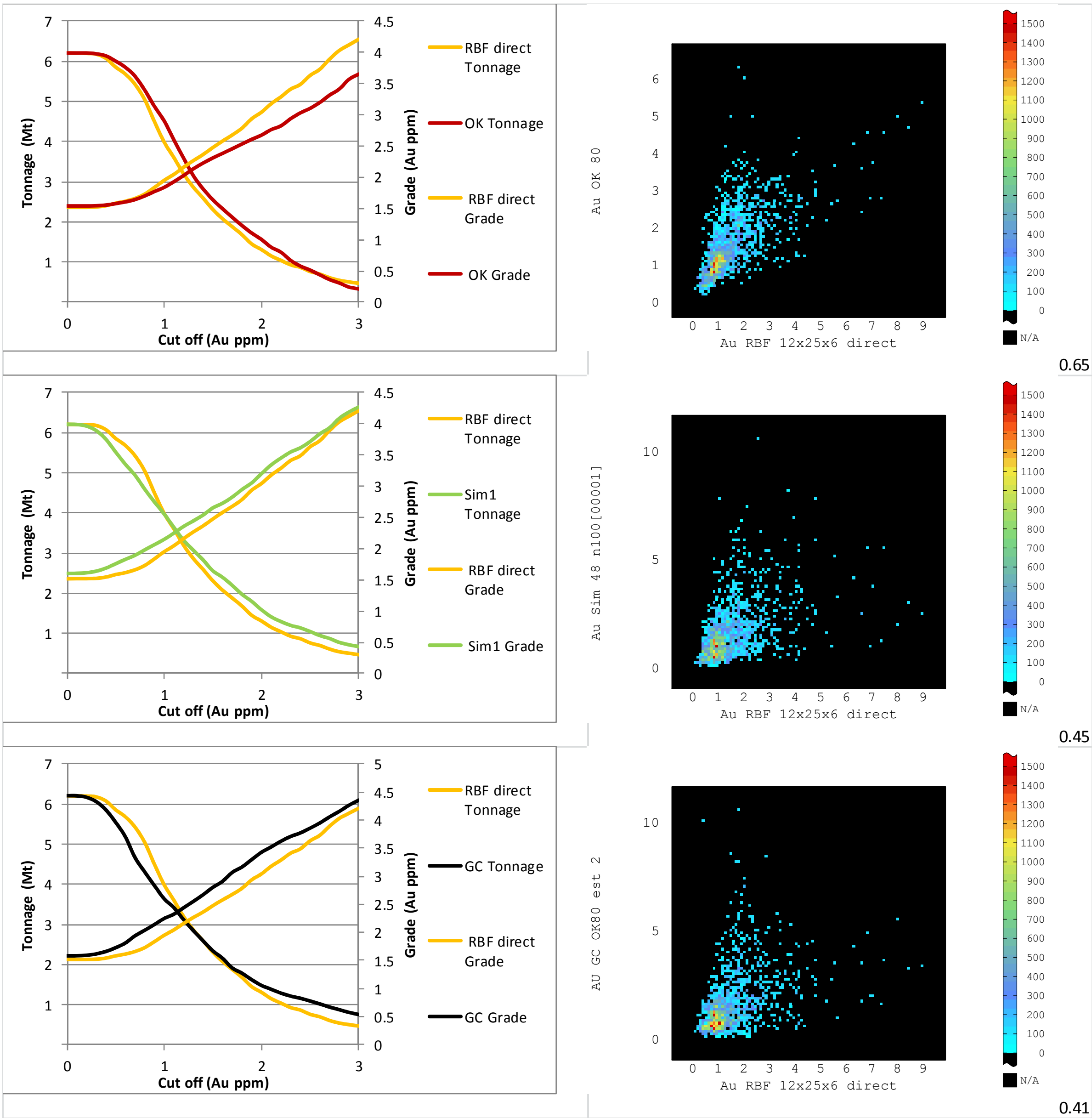

Figure 5: Radial Basis Function Direct compared with Ordinary Kriging 80, Sim1 and Grade Control (Reality)

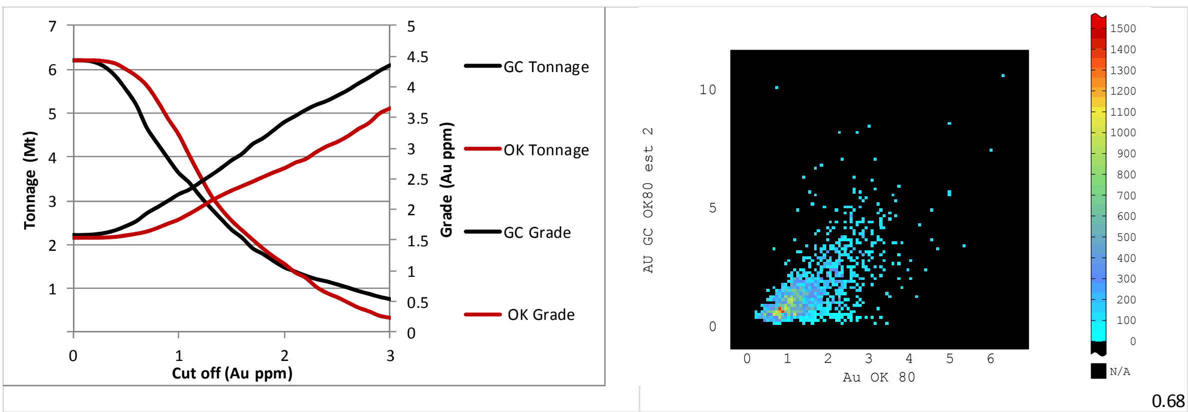

Figure 6: Ordinary Kriging compared with Grade Control (Reality)

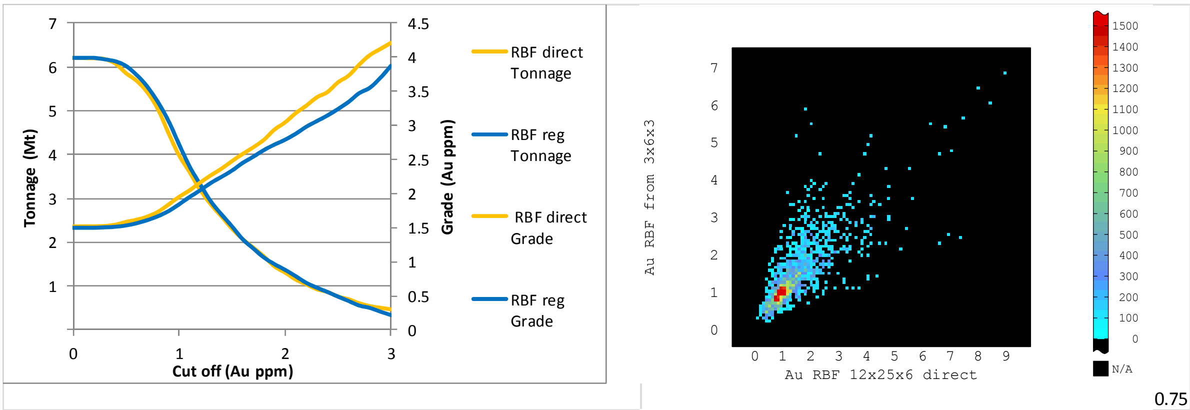

Figure 7: Radial Basis Function Regularised compared with Radial Basis Function Direct

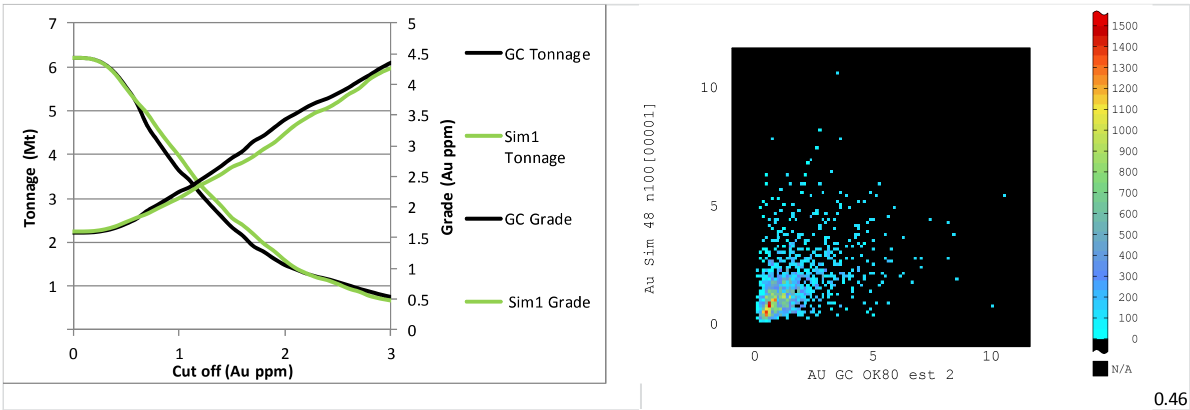

Figure 8: Grade Control (Reality) compared with Sim1

Observations

Observations from Figures 4 - 8 are:

1. RBF Reg is a reasonably close match to OK both globally and locally, with grade and tonnage curves closely matching and with a high block-to-block correlation of 0.94.
2. RBF Reg does not compare well with Sim1, globally or locally. This is expected as a simulation should have greater variability compared to the smoothing inherent in the OK estimator.
3. RBF Reg, when compared with RBF Direct shows that, although global tonnages compare well, global grades diverge as cut off increases. This is due to the absence of block support correction when using RBF Direct. Locally, there is a reasonable correlation at 0.75 but still significant scatter.
4. Given the similarities between OK and RBF Reg, observation 3 also applies to the RBF Direct and OK comparison.
5. RBF Direct and GC do not compare particularly well, either globally or locally. Locally, the correlation is very poor at 0.41 with a large amount of scatter.
6. GC compares well to Sim1 globally with well-matched grade and tonnage curves. It compares poorly on a block-by-block basis, giving a correlation of only 0.46 and a very large amount of scatter. This outcome is expected because we know simulations reproduce global variability and will produce more realistic grade tonnage curves at the expense of local block accuracy.
7. RBF Reg compared with GC (Reality) shows significant differences both globally and locally, although they are similar at a 0 ppm Au cut off. This is not unusual when comparing resource models based on wide-spaced drilling with grade control models based on dense data. With positively skewed distributions such as gold, wide-spaced drilling will often under-represent the true distribution of high grades. The local correlation is 0.66.
8. Similar comments as for observation 7 apply to OK compared with GC (Reality) although OK has a slightly better local correlation at 0.68 compared to 0.66. The key point is that OK from exploration data is still the best local estimator compared to reality (GC model).

Conclusion

For this dataset it was observed that, when used as a point estimator and regularised into blocks RBF performs equally as well as Ordinary Kriging. When used as a direct block estimator, RBF is inferior to Ordinary Kriging on a local basis as it does not properly account for block support. RBF in general is not a substitute for simulation as it does not reproduce true block variability.

The results show that RBFs can be used in a similar manner to Ordinary Kriging with the skewed data sets that are typical of gold deposits without further transformation or manipulation to normalise the distribution.

References

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