A comparison of procedures for determining the state parameter of silt-like tailings

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ABSTRACT: After recent failures related to tailings storage facilities, the assessment of static liquefaction has become a major subject of interest in the mining industry. The standard procedures to evaluate global stability under liquefied conditions combines limit equilibrium with trigger analyses, which have fallen short -in some cases- in putting geotechnical risk off the table. Thus, deformation modelling incorporating high-end numerical procedures is often called in, requiring the calibration of a large number of material parameters that entails uncertainty and condition the robustness of the approach. One of the most challenging parameters to calibrate in tailings deposits is the state parameter for silt-like tailings that fall in the intermediate region between clays and sands. This work describes the application of the Shuttle & Jefferies (2016) methodology to estimate the state parameter along CPTu soundings using numerical cavity expansion element tests using Norsand, and a comparison with a more conventional approach like the one by Been & Jefferies (2016). The spatial distribution of the state parameter, when determined using this procedure and fed into a dam section, allows for defining regions of similar expected behaviour within the tailings body. An application for a tailings storage facility is illustrated, the results of two screening methods are compared and discussed.

1 INTRODUCTION

Mine tailings are a by-product of rock-crushing that consist mainly of sand/silt size particles and are generally deposited as a slurry into storage facilities. The lack of post-deposition compaction mechanisms and the electrical interaction among the finer particles generally entail loose in-situ arrangements; which, combined with full saturation and static/dynamic rapid loading, can lead to the generation of pore pressures that eventually liquefy the material. Thus, quantifying the material contractiveness is of paramount importance for practical engineering purposes.

Sand-like tailings have been described under the framework of Critical State Soil Mechanics (e. g. Bedin et al 2012, Been 2016, Been & Jefferies 2016). Within this framework, the state parameter ψ, proposed by Been & Jefferies (1985), has been widely used to quantify granular material tendency to contract or dilate based on the distance from the current $p' - e$ state to the Critical State Line (CSL).

When the silt content is high, however, tailings behaviour combines aspects from clay-like and sand-like materials; it resembles clay behaviour in terms of low hydraulic conductivity, and sand behaviour in terms of having the strength controlled by particle contacts; entailing dilative/contractive behaviour depending on the void ratio e and mean effective pressure p'; and challenges in obtaining undisturbed samples, as the unavoidable densification induced by handling and transportation can lead to a dramatic change in the material response (Been 2016, Been & Jefferies 2016).

The impossibility of obtaining undisturbed samples has led to the development of correlations and methods to estimate ψ of tailings from CPTu measurements (e.g. Robertson 2012); however, most of these correlations were mainly developed and calibrated for sand-like materials and are sometimes not fully representative of silt-like tailings. Some correlations (e.g. Been et al 2012, Dienstmann et al 2018), while developed for tailings, are empirical in nature and thus require extensive calibration for a particular tailings dam site.

Shuttle and Jefferies (2016) chose a more fundamental approach where cavity expansion theory is recalled to establish a correlation between the tip resistance of CPTu and the state parameter. In this paper, this approach is labelled S&J and is applied to evaluate the spatial distribution of ψ of an upstream raised tailing storage facility based on a set of CPTu tests. Results are contrasted with the more classical approach by Been & Jefferies (2016), labelled B&J here, and the differences between both methodologies are evaluated and commented.

2 SHUTTLE & JEFFERIES METHODOLOGY

2.1 *Description*

Shuttle & Jefferies (2016) postulate that the CPTu tip resistance can be expressed as a function of the limit pressure of a soil under spherical cavity expansion and a mapping factor that essentially relates the sphere with the cone geometry. The methodology has the aim to compute parameters k and m (to be introduced below), for silty materials, allowing for estimating ψ from CPTu measurements.

The procedure can be summarized into four main steps. First, the NorSand constitutive model (Jefferies 1993) is used to compute the limit radial cavity effective stress p'_{lim} for different combinations of effective initial mean pressure p'_0, state parameter ψ_0 and rigidity index I_r. This is done using a 1D large-strain finite element open access code, referred to as CPTWidget (Shuttle & Jefferies 2016) and simulating a spherical cavity expansion to a radial strain $\epsilon_{rr} = 500\%$. Thus, for each combination of $p'_0 - \psi_0 - I_r$, a limit effective pressure p'_{lim} is obtained (Fig 1).

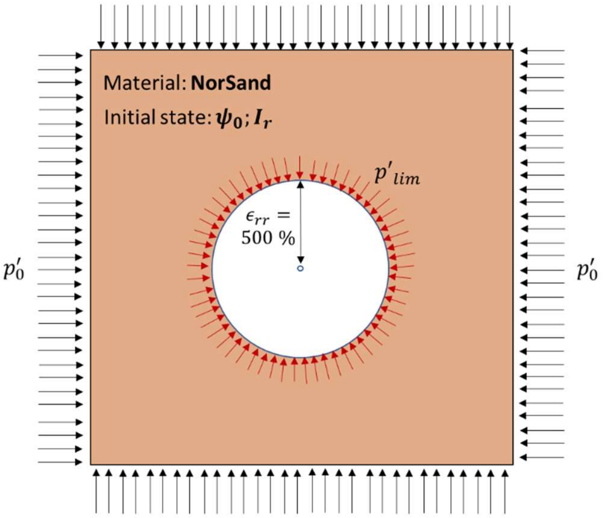

Figure 1. Scheme to compute the cavity expansion limit effective stress (p'$_{\text{lim}}$) using NorSand and a large strain 1D FEM code (Shuttle & Jefferies 2016).

Then, p'_{lim} is mapped to the cone geometry using a scaling factor C_Q and the pore pressure u obtained from the numerical radial expansion is added to compute a total tip resistance as

$$q_t^* = C_Q p'_{lim} + u \tag{1}$$

The mapping factor is $C_Q = 3.2\exp(-2.4\psi)$, adjusted from calibration chamber tests in sands.

In a third step, the normalized cone tip resistance and excess pore pressure are computed as

$$Q_p = (q_t^* - p_0)/p'_0 \tag{2}$$

$$B_q = (u_c - u_0)/(q_t^* - p_0) \tag{3}$$

where p_0 is the total initial mean pressure, u_0 is the initial pore pressure, and u_c is the shear-induced pore pressure numerically computed with CPTWidget.

In the last step, a dimensionless grouping is defined as $Q_p\left(1 - B_q\right) + 1$, and it is related to CPTu field measurements by

$$Q_p\left(1 - B_q\right) + 1 = (q_t^* - u_c)/p'_0 \sim (q_t - u_2)/p'_0 \tag{4}$$

where q_t^* is approximated with the cone tip resistance q_t and u_c with the cone pore pressure u_2. Therefore, there is a unique and direct relationship between the numerical results and the cone measurements, where the bridge is the dimensionless soil strength grouping. The state parameter can then be calculated using

$$m\psi = -\ln\left[\left(Q_p\left(1 - B_q\right) + 1\right)/k\right] \tag{5}$$

where k is the normalized tip resistance at $\psi = 0$ and m is the slope of the best-fit straight line in the semi-log space.

In summary, the method uses a state-parameter dependent constitutive model to numerically compute several dimensionless grouping soil resistances for different initial conditions $p'_0 - \psi_0 - I_r$, plot them in the $\ln\left[Q_p\left(1 - B_q\right) + 1\right]$ vs ψ_0 space; and determine parameters k and m. Once the procedure is complete, CPTu measurements are input in eqn. (4) to estimate $Q_p\left(1 - B_q\right) + 1$ which is the input in eqn. (5) to back calculate ψ.

It must be noted that the package provided by the authors automatizes much of the aforementioned steps, such that the user needs just to: i) calibrate NorSand; ii) run CPTWidget; iii) get results and manually adjust the k and m parameters (Shuttle & Jefferies 2016).

3 APPLICATION TO A CASE STUDY

3.1 *Data sources*

This study is based on data from a field and laboratory testing program performed on an upstream-raised tailings storage facility. Field tests comprise CPTu soundings at different locations within a flank of the dam body -toe, mid-berm and crest-; while laboratory tests entail conventional physical and mechanical tests including specific gravity, minimum and maximum dry densities, particle size distribution and isotropically consolidated drained / undrained triaxial compression tests (CIDC|CIUC) performed on two different reconstituted tailing samples.

Void ratios were determined using the freezing method at the end of the triaxial tests -as suggested by Been (2006)-, which allows to expect good accuracy of this variable.

3.2 *Material*

Two tailings samples were studied. Sample A (fine tailings) has 40 % sand, 50 % silt and 10 % clay size particles; minimum and maximum dry densities are 12.2 and 21.0 kN/m^3 respectively and a specific gravity of 2.75; this results in equivalent minimum and maximum void ratios of 0.28 and 1.21, respectively.

Sample B (coarse tailings) has 77 % sand, 19 % silt and 4 % clay size particles; minimum and maximum dry densities are 12.9 and 22.3 kN/m^3, and specific gravity is 2.79; this results in equivalent minimum and maximum void ratios of 0.21 and 1.10. An illustration of both samples is shown in Fig 2.

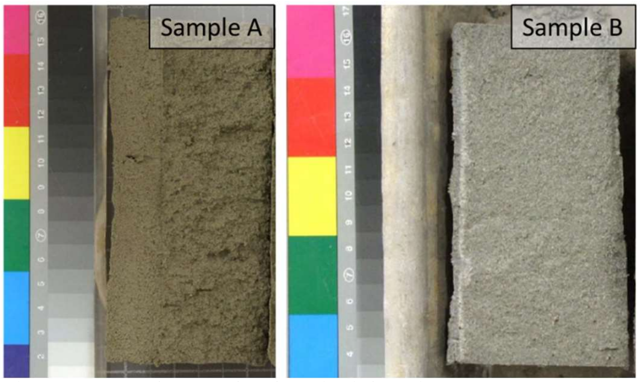

Figure 2. Sample A and B for triaxial testing.

3.3 *NorSand calibration*

NorSand constitutive model (Jefferies 1993, Been & Jefferies 2016) is calibrated for the two samples using triaxial test data (CIDC and CIUC). A summary of the calibrated parameters is shown in Table 1. The reader is referred to Shuttle & Jefferies (2016) for the definition of the material parameters of Norsand. Details of the calibration are shown in Figure 3.

Table 1. NorSand parameters.

Parameter	Sample	
	A	B
Γ	1.02	1.12
λ_{10}	0.170	0.175
M_{tc}	1.29	1.24
N	0.00	0.00
χ	3.50	2.00
H_0	50	35
H_ψ	350	150
ν	0.15	0.15
I_r	100	100

The following observations can be made: i) remarkable agreement is obtained for the CIUC test, especially in terms $p' - q$ and $q - \epsilon_a$; ii) for the CIDC tests, a good agreement with the data is achieved, with minor limitations on reproducing the volumetric strains; iii) the calibrated parameters for both samples are very similar, specially the three that define the CSL (Γ, λ_{10} and M_{tc}).

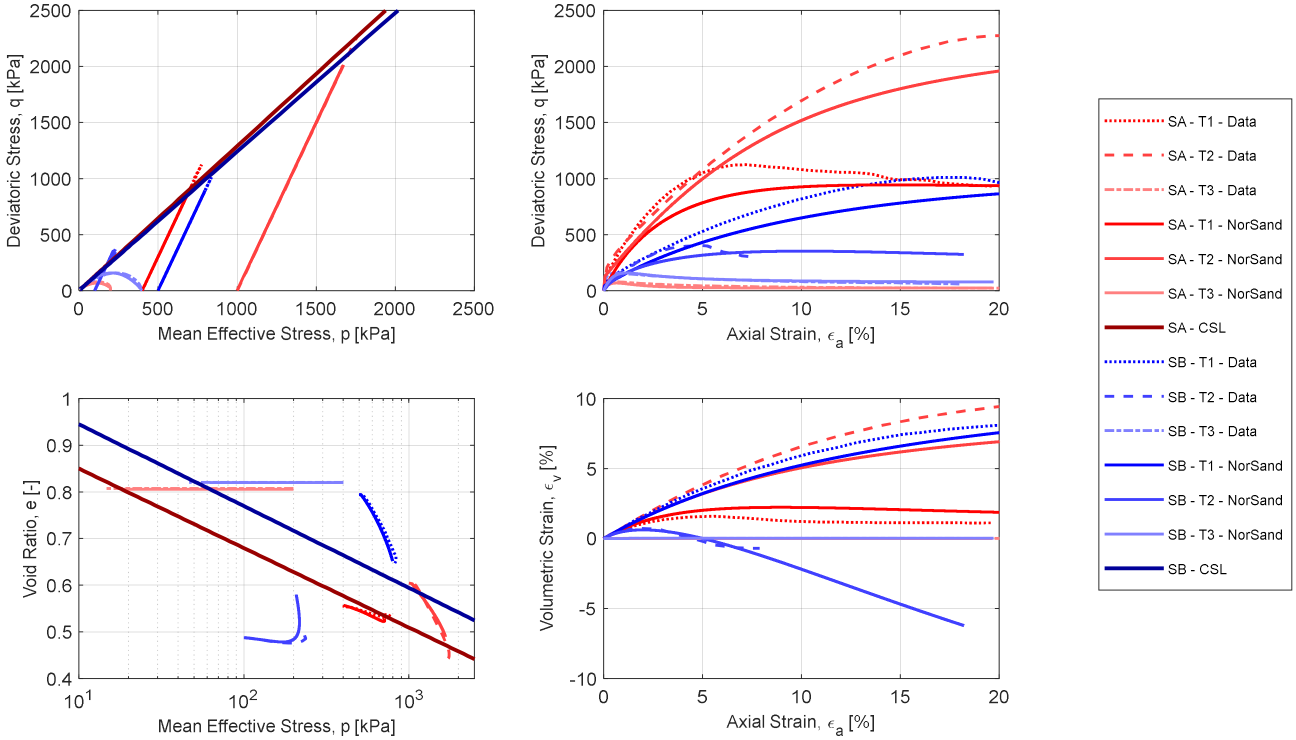

Figure 3. Comparison between laboratory data and NorSand calibration for Samples A and B.

3.4 *Determination of parameters k and m*

NorSand calibration for both samples are combined with drained and undrained cavity expansion analyses using CPTWidget. For the drained analyses, a total of 48 different initial states are defined, by combining rigidity indexes $I_r = 50|100|150|200$, initial mean effective pressure $p'_0 = 100|500$ kPa and initial state parameter $\psi_0 = -0.05|0.00|0.05|0.10|0.15|0.20$. For the undrained analyses, a total of 24 combinations are modelled, using the same initial mean effective stresses and state parameter but only analysing $I_r = 50|150$ as p'_{lim} for undrained cavity expansion is ideally independent of soil stiffness.

Results for both samples are presented in terms of dimensionless penetration resistance as a function of $p_0' - \psi_0 - I_r$ (Fig 4). It is shown that i) a reasonable linearity is obtained in the semi-log space, ii) the penetration resistance is normalizable by the initial effective pressure; iii) rigidity index play a minor role.

For Sample A, the drained case is approximated using $k = 26.0$ and $m = 7.2$, and the undrained case by $k = 10.0$ and $m = 11.0$. For sample B, $k = 22.5$ and $m = 5.8$ for the drained case, and $k = 9.0$ and $m = 10.0$ for the undrained case.

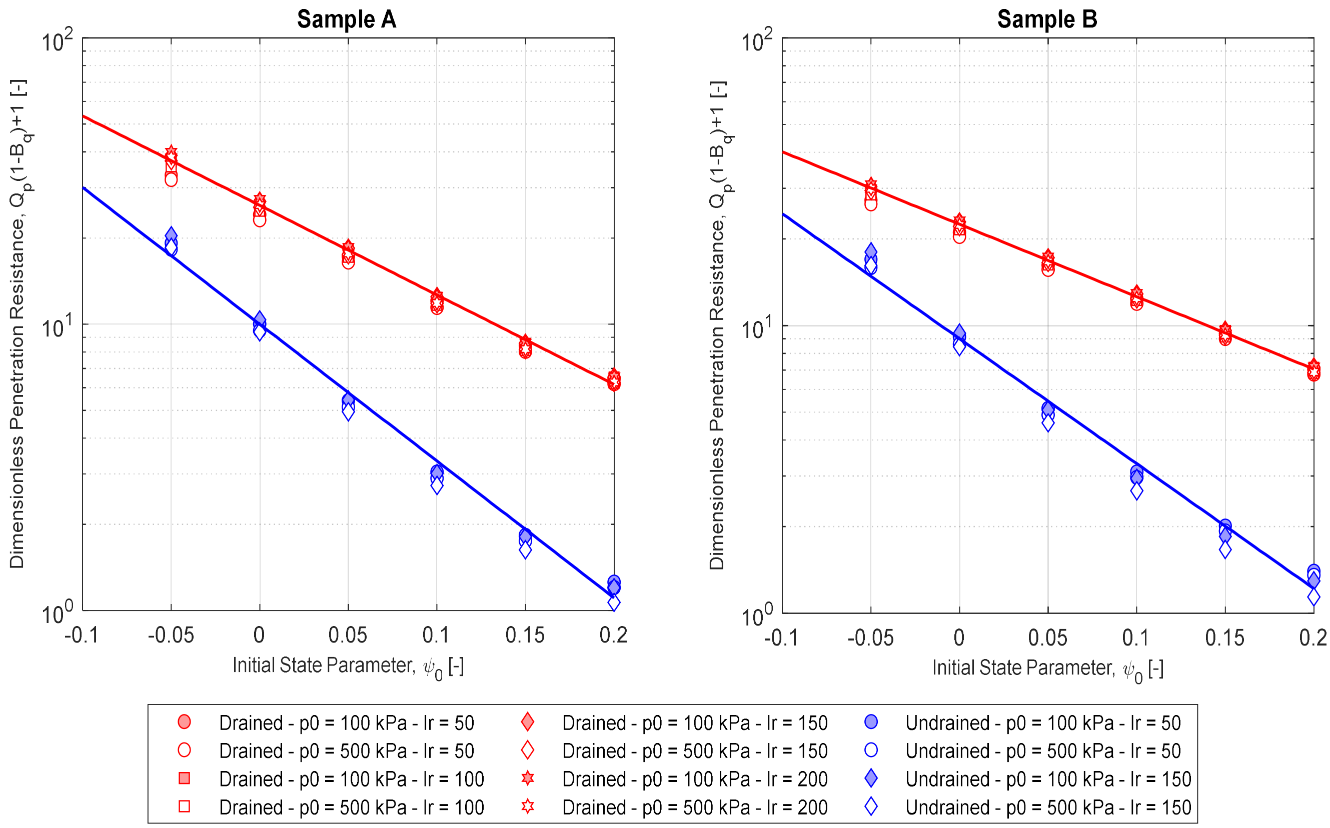

Figure 4. CPTWidget results. Dimensionless penetration resistance as a function of initial state parameter. Samples A and B.

4 DETERMINATION OF STATE PARAMETER FROM CPTU DATA

4.1 *Procedure*

The analyses using CPTWidget define k and m for drained/undrained expansion of the two samples (four sets of parameters). These four sets are then used as the basis for estimating ψ in the field using CPTu data, depending on the tailings characteristics and whether the cone penetration is drained or undrained.

In this study, the distinction between the sandy and silty tailings is made based on the I_c value proposed by Been & Jefferies (2016); those with $I_c > 1.8$ are identified as sandy-silt/silt tailings represented by sample A, and those with $I_c < 1.8$ are identified as sand/silty-sand tailings represented by sample B.

The distinction between drained and undrained penetration is made based on the absolute value of B_q, also proposed by Been & Jefferies (2016). Zones with $|B_q| < 0.02$ are considered as drained, while those showing $\left|B_q\right| > 0.02$ are considered as undrained. By a continuous screening of each CPTu sounding, the state parameter is estimated depending on which one of the four cases applies (Table 2).

Table 2. Adopted k-m parameters to compute ψ from CPTu.

	Sandy Silt (Sample A)	Silty Sand (Sample B)
	$I_c > 1.8$	$I_c < 1.8$
Drained, $\lvert B_q \rvert < 0.02$	$k = 26.0$ $m = 7.20$	$k = 22.5$ $m = 5.80$
Undrained, $\lvert B_q \rvert > 0.02$	$k = 10.0$ $m = 11.0$	$k = 9.0$ $m = 10.0$

4.2 *Individual soundings*

Results of interpreting the state parameter ψ along the CPTu soundings using Shuttle & Jefferies (2016) are compared with the method described in Been & Jefferies (2016). The procedure is enhanced with a frequency analysis of ψ along the soundings.

As an example, results of both methods for a CPTu sounding located close to the TSF crest are shown (Fig 5). It is observed that: i) a fairly good agreement is achieved between both methods; ii) the dispersions are similar, but S&J, in general, predicts a higher mean value of ψ; iii) being close to the border, tailings are classified as dilative by B&J (mode $\psi = -0.10$) and contractive by S&J (mode $\psi = -0.04$), a difference which is important in practical design.

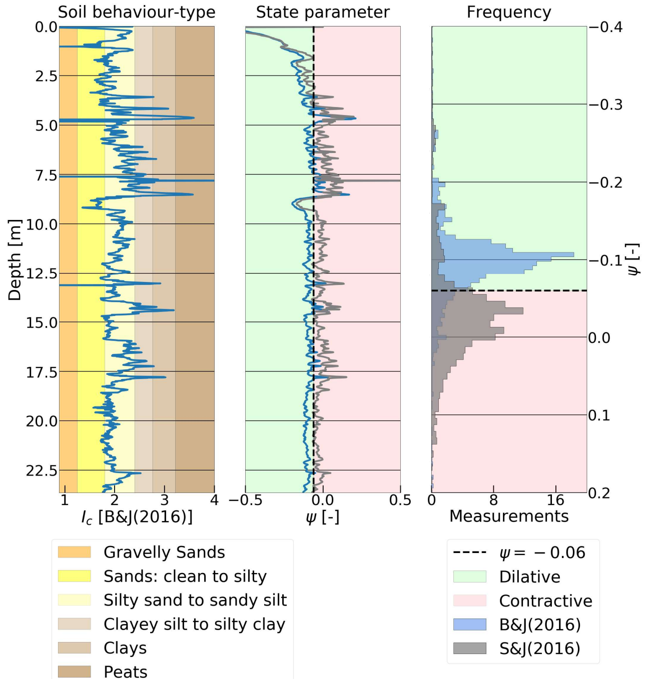

Figure 5. Interpreted I_c and ψ for a CPTu near the TSF crest.

4.3 *Spatial variability on a cross-section*

In the case study shown here, the state parameter is interpreted for eight CTPu soundings, using the two aforementioned methods: Shuttle & Jefferies (2016) and Been & Jefferies (2016). All the results are placed within a TSF representative cross-section (Fig. 6). Due to the large size of the TSF, there are significant portions of the tailings body that have no data, a challenge that emphasizes the need for extracting the most of each in-situ data available.

It must be noted that only values of state parameter referring to contractive behaviour ($\psi > -0.06$) are shown; therefore, blank spaces between values must be interpreted as dilative material.

The following observations can be made: i) there are soundings for which the material classification dramatically shifts from dilative to contractive (such as CPTu-D, E and H ; ii) the areas close to the toe and the mid berm show the most favourable conditions and those close to the crest show the most unfavourable conditions, with higher values of ψ.

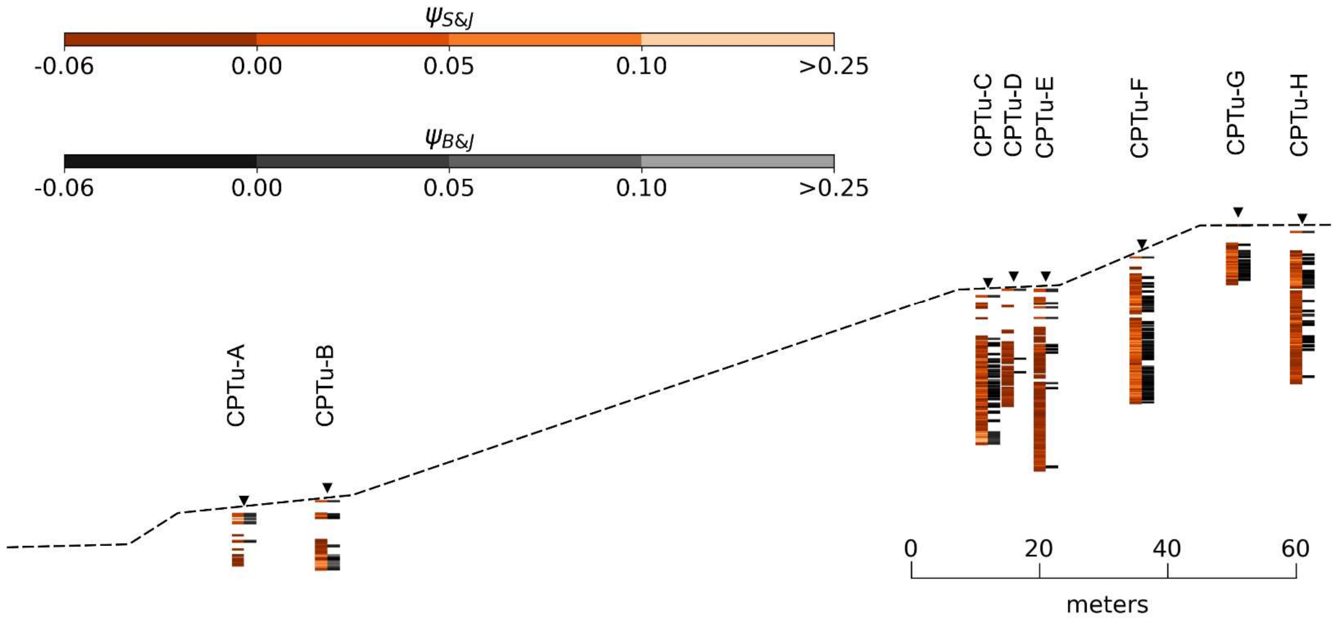

Figure 6. Distribution of ψ along a TSF cross-section. Comparison between Been & Jefferies (2016) and Shuttle & Jefferies (2016).

5 DISCUSSION

5.1 *State parameter*

Considering that both Been & Jefferies (2016) and Shuttle & Jefferies (2016) methods entail state parameters with similar dispersions fluctuating near the dilative/contractive threshold value ($\psi = -0.06$), it is of great interest to quantify the difference between both methods. The state parameter variation between S&J and B&J is defined as

$$\Delta\psi = \psi_{S\&J} - \psi_{B\&J} \tag{6}$$

The difference for each CPTu sounding is interpreted and plotted in box type graphs (Fig. 7), where the box represents the 50% of $\Delta\psi$ and the position of the median (inside line); the upper and lower quartiles are indicated by outside lines. It is observed that: i) the state parameter from S&J is systematically higher than B&J; ii) $\Delta\psi$ median values are all positive, with values increasing from toe to crest; iii) the maximum $\Delta\psi$ can be up to 0.13, which is quite significant and has serious implications in the dam design.

5.2 *Contractive behaviour, comparison*

Figure 8 shows a comparison between the prediction of contractive behaviour of Shuttle & Jefferies (2016) and Been & Jefferies (2016) in terms of the ratio of the state parameters that reached the contractive region ($\psi > -0.06$).

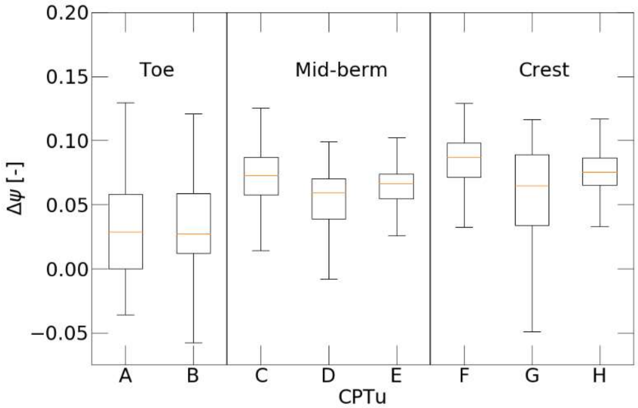

Figure 7. Variation of the state parameter increment between Shuttle & Jefferies to Been & Jefferies.

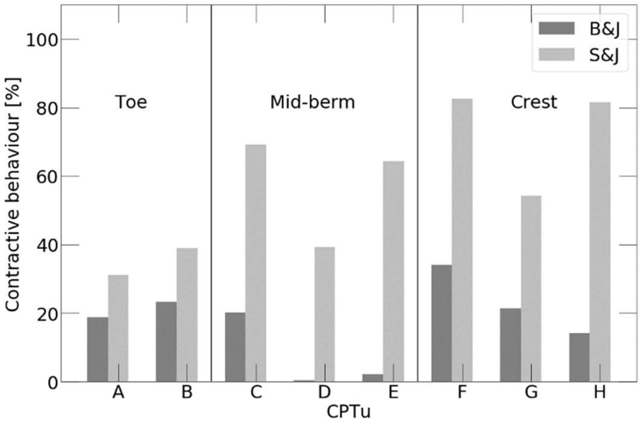

Figure 8. Comparison of contractive behaviour between Shuttle & Jefferies and Been & Jefferies.

As shown in Fig. 8, the state parameter estimated employing S&J results in higher contractiveness than values obtained from B&J. A remarkable difference is observed in CPTu-D and E, which results in predominantly dilative behaviour by B&J in contrast to 40% and 65% of contractiveness reached by the S&J procedure.

Being based on a more fundamental understanding of the physics involved in cone penetration and having a calibration procedure which relies in the constitutive model built around the concept of state parameter, it is concludes that the Shuttle & Jefferies (2016) procedure is more reliable for tailings than empirically-based procedures like Been & Jefferies (2016). This statement, however, must be critically judged in the context of each particular project involving the analysis of tailings in undrained shear.

6 CONCLUSIONS

Characterization of the state parameter of silt-like mine tailings can be performed in the framework of Critical State Soil Mechanics and employing the state parameter ψ. Shuttle and Jefferies (2016) employed cavity expansion theory to establish a correlation between the tip resistance of CPTu and the state parameter. Their procedure is briefly explained in this paper, and its result is compared with a more empirical approach by Been & Jefferies (2016). A case study is presented where 8 CPTu soundings were screened using both procedures. It is observed that the state parameter using Shuttle & Jefferies (2016) procedure is systematically higher than Been & Jefferies (2016); and the maximum difference can be up to 0.13, which is a quite significant value.

It is concluded that the procedure by Shuttle & Jefferies (2016) is based on a more fundamental understanding of the physics involved in cone penetration and uses a constitutive model built around the concept of state parameter, making it more reliable for silt-like tailings than empirically-based procedures.

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