PROBABILISTIC METHODS FOR SLOPE ANALYSIS AND DESIGN

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1 INTRODUCTION

Probabilistic methods combined with risk assessment are a better way to assess slope design in open pit mines compared to deterministic methods. These methods are suitable for use on evaluation of risk or when there is uncertainty in the input parameters.

Probabilistic analyses require more computer power than deterministic analysis. In many case a probabilistic analysis requires ten to thousands more computer resources than an equivalent deterministic analysis. Methods like Monte Carlo simulation (MC) may require thousands of analyses depending on the number of variables considered in the model. Other methods like First Order Second Moment (FOSM) or Point Estimate Method (PEM) and may require tens to hundreds of analyses.

Monte Carlo simulation is applied routinely today on simple analyses like wedge stability or limit equilibrium analysis; current computers can carry thousand of analysis in a relatively short period of time. This is not the case when more complex models are built like 3D models at mine scale including complex mining sequences, or dynamic analysis of a 3D model. Large scale models can run for hours even in fast computers, where the Monte Carlo method is not an option other alternative methods should be used.

This paper compares four different methods and presents the equations required to use a Modified Point Estimate Method (mPEM) presented by Harr (1989). The methods are compared using simple examples in the paper.

Recommended probabilities of failure for open pit design are also presented.

2 PROBABILISTIC METHODS USED IN STABILITY ANALYSIS

Stability analysis is the main consideration used to define the geometry of a slope. It is possible to identify 3 scales of analysis used to define the pit slope:

- Bench scale
- Inter-ramp scale
- Overall scale

A discussion about the recommended probability of failure (P_f) is given in the next section. Methods to assess P_f are discussed below.

2.1 METHODS OF ANALYSIS

2.1.1 Monte Carlo Simulation

MC is a simple method that evaluates the problem many times using random input parameters. For each analysis P_f is computed by evaluating a target function. The method requires a definition of failure to be established prior to the analysis being undertaken. Examples include deformation at a specific point larger than a predefined value, Factor of Safety (FS) < 1.0 or Reliability Index (β) less then β_{crit}.

Some issues to keep in mind when MC is used:

- Consider the proper distribution that best represents the variables included in the model e.g. normal distribution might produce negative numbers that have no physical meaning like negative friction or cohesion. In these cases it is better to use a log normal distribution.

- Where appropriate, the model should include correlation between variables. In these cases independent generation of random parameters is not appropriate. For correlated parameters equations 1 to 5 may be used.

$$N_1 = \sqrt{-2lnR_1}\cos(2\pi R_2) \quad \textbf{(1)}$$

$$N_2 = \sqrt{-2lnR_1}\sin(2\pi R_2) \quad \textbf{(2)}$$

$$N_2^* = N_1\rho_{XY} + N_2\sqrt{1-\rho_{XY}^2} \quad \textbf{(3)}$$

$$x_1 = \mu_x + N_1\sigma_x \quad \textbf{(4)}$$

$$y_1 = \mu_y + N_2^* \sigma_y \tag{5}$$

Where

R_1 and R_2 are independent random numbers between 0 and 1 with uniform distribution.
N_1 and N_2 are independent random numbers normally distributed with average 0 and standard deviation (σ) 1.
N_2^* is the random number correlated with N_1.
$\rho_{xy}$is the correlation coefficient between random variables X and Y.
μ_x, σ_x, μ_y, σ_y are average and standard deviation of variables X and Y.

- Run enough simulations to ensure the control variable (deformation, FS or Reliability Index (β)) is accurate enough. If the error on the probability of failure is defined by α according Figure 1. It is possible to estimate the number of simulations required using Equation 6:

$$n = \left(\frac{d}{\alpha}\right)^2 \frac{1-p}{p} \tag{6}$$

Where:

n Number of simulations
d Normal standard deviate estimated form Table 1
α Acceptable error in the analysis to assess the probability of failure
p Probability of failure

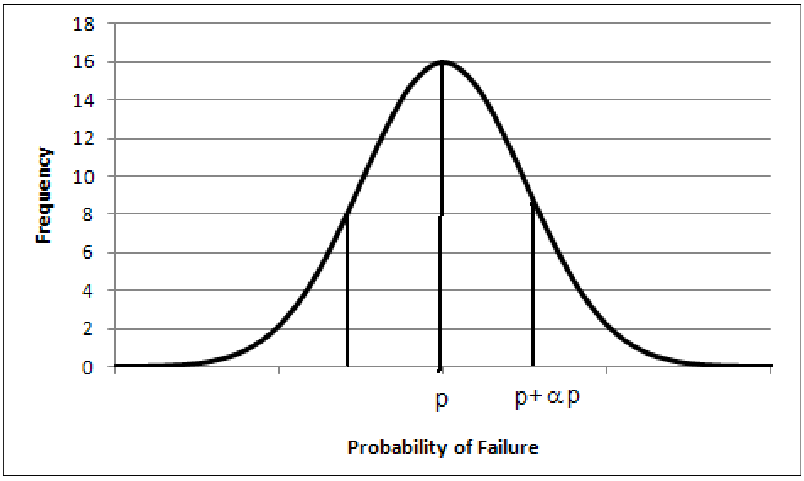

Figure 1: Distribution of Probability of Failure

Table 1: Normal Standard Deviate

Percentage of Confidence (%)	Normal Standard Deviate (d)
80	1.28
85	1.44
90	1.64
95	1.96
99	2.57

Two issues should be noted in respect of Equation (6): the number of Monte Carlo simulations required is not a function of the number of variables involved in the problem. Second, probability of failure is not known in advance, so in order to use Equation 6 the probability (p) should be estimated. This may seem a big drawback in having to estimate the number of iterations required, but in general, an engineer will be able to assess the probability of failure required for a specific design.

Monte Carlo simulation is popular on simple problems were a large number of simulation can be run in minutes. It doesn't require an assumption about the distribution of the target function. The method is not suitable for complex problems (3D models at mine scale, dynamic analyses) where one simulation takes hours and in some cases days.

Example 1: The probability of failure for a slope is estimated to be p=0.05, calculate the number of Monte Carlo simulations required to assess the probability in a range ± 10% around the estimated p with 95% of confidence.

In this case d=1.96, α=0.1 and p=0.05

$$n = \frac{1.96^2}{0.1^2}\frac{(1-0.05)}{0.05} = 7299$$

Example 2: The stability for planar failure indicated in Figure 2 can be assessed using Equations 7 to 10.

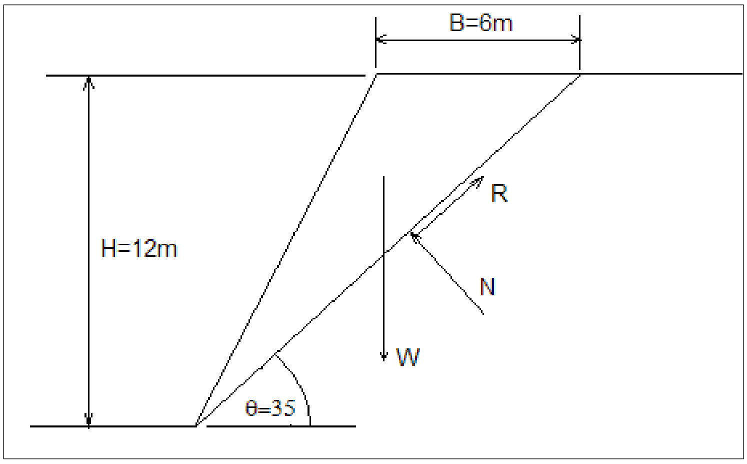

Figure 2: Planar Slope Failure.

$$W = \frac{1}{2}\gamma HB \quad \textbf{(7)}$$

$$N = W\cos\theta \quad \textbf{(8)}$$

$$R = cL + W\cos\theta\tan\phi \quad \textbf{(9)}$$

$$FS = \frac{cL+W\cos\theta\tan\phi}{W\sin\theta} \quad \textbf{(10)}$$

Note that in the previous equations: L is the length of the joint, c is the cohesion, ϕ is the friction angle and γ is the unit weight. The unit weight, the force due to cohesion and the friction were assumed to be variables with log normal distribution, the values used in the analysis are indicated in Table 2. It was assumed that there is no correlation between friction and cohesion.

Table 2: Parameters

Variable	Average Value	Variance (%)	Standard Deviation	Distribution
Unit Weight [kN/m^3]	27	10	2.7	Log normal
Cohesion [kN]	9.6	40	3.84	Log normal
Friction (tan ϕ)	0.577	12	0.069	Log normal

The probability of failure of the slope shown in Figure 2 was calculated using n=1000 Monte Carlo simulations, the value obtained is P_f =0.215, E[FS]=1.126 with a Standard Deviation σ=0.158. To assess the range of probability of failure with a confidence of 90% Equation 6 is used, solving for α□ where: d=1.64 n=1000 p=0.215

$$\alpha = 1.64\sqrt{\frac{(1-0.215)}{(1000)(0.215)}} = 0.060$$

There is a 90% chance that the actual probability of failure is in the range:

[0.215 - 0.215 x 0.060, 0.215 + 0.215 x 0.060] = [0.202, 0.228].

Figure 3 shows the variation of the probability of failure along with the upper and lower limits and the number of Monte Carlo simulations.

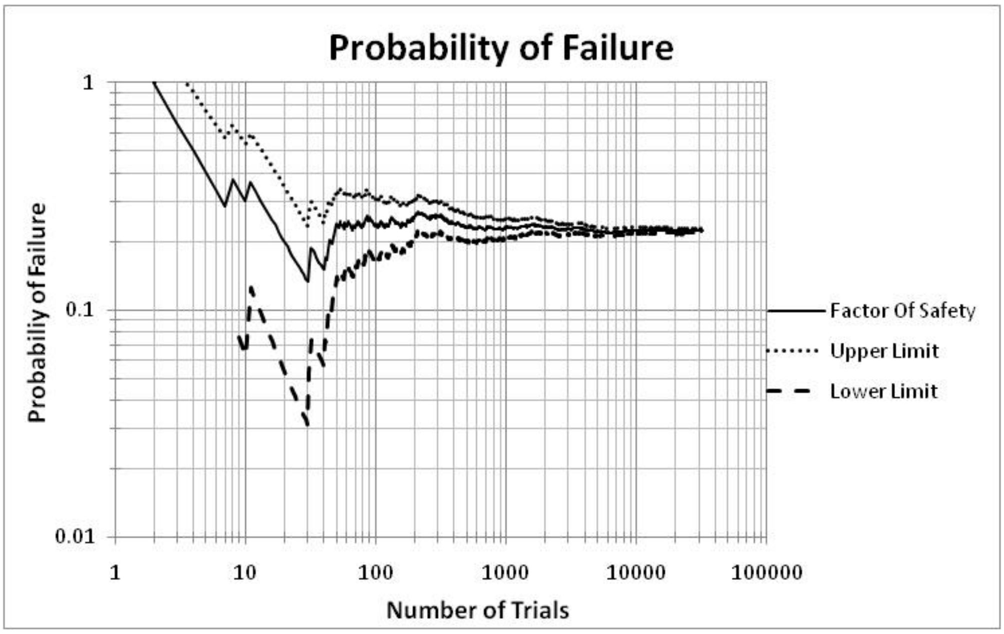

Figure 3: Probability of Failure vs Number Monte Carlo Simulations

The three methods are shown in the following sections are part of a family of probabilistic methods based on an assessment of the mean values and standard deviation (or Variance) of the target function. These methods do not assess the full distribution of the target function therefore an assessment about the expected distribution of the target function should be made (normal, lognormal, u other).

2.1.2 First Order Second Moment Method (FOSM)

This method is based on a Taylor's series expansion on the target function ($f(x_i)$) about some point. The method provides an estimate of the moments (mean and standard deviation) of the target function based on moments of the N inputs variables.

The Taylor series expansion for a multi variables function $f(x_i)$ retaining only the linear components is shown in Equations 11 and 12:

$$y = f(x) = f(x_1, x_2, \dots, x_N) \quad \textbf{(11)}$$

$$y = f(x) = f(\overline{x_i}) + \sum_{i=1}^{n}(x_i - \bar{x}_i)\frac{\partial y}{\partial x_i} \quad \textbf{(12)}$$

From the previous equations it is possible to prove (US ACE, 1997) the expected value of y (E[y]) and the Variance of y (Var[y]) can be calculated as shown in Equations 13 and 14:

$$E[y] = f(\bar{x}_i) + \frac{1}{2}\sum\frac{\partial^2 y}{\partial x_i \partial x_j}Cov(x_i, x_j) \quad \textbf{(13)}$$

$$Var[y] = \sum\left[\left(\frac{\partial y}{\partial x_i}\right)^2 Var[x_i]\right] + 2\sum\left[\frac{\partial y}{\partial x_i}\frac{\partial y}{\partial x_j}Cov(x_i, x_j)\right] \quad \textbf{(14)}$$

It is common not to use the expression with Covariance in Equation 13. (US ACE, 1997). Equations 13 and 14 require the evaluation of the derivative of the target function y with respect to the variables. This can be estimated using Equation 15.

$$\frac{\partial y}{\partial x_i} \approx \frac{f(\bar{x}_1, \dots, \bar{x}_i + \sigma_i, \dots, \bar{x}_N) - f(\bar{x}_1, \dots, \bar{x}_i - \sigma_i, \dots, \bar{x}_N)}{2\sigma_i} \quad \textbf{(15)}$$

This method requires 2N+1 evaluations of the target function. In some cases where target function is not known the evaluation is done via numerical analyses, this imply that it will be required 2N+1 analyses; for instance, slope stability analysis using limit equilibrium methods and considering material properties as variables will require several analyses to evaluate Equation 15.

The method is applied to the planar failure shown in Figure 4. In this case, it was considered that the joint dip is variable; a bolt has been added to improve stability. The variables assumed in the problem are shown in Table 3. No correlation was assumed between the variables.

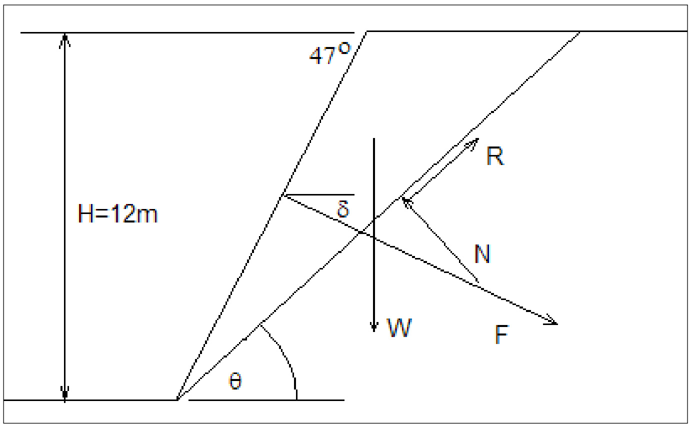

Figure 4: Planar Failure Including Bolt Support

Table 3: Parameters

Variable	Symbol	Average Value	Variance (%)	Standard Deviation	Distribution
Unit Weight [kN/m^3]	W	27	10	2.7	Log normal
Cohesion [kPa]	C	9.6	40	3.84	Log normal
Friction (tanϕ)	ϕ	0.577	12	0.069	Log normal
Joint dip (θ)	θ	35º	10	3.5º	Normal
Bolt Force [kN]	F	70	10	7	Normal
Bolt Inclination	δ	10º	20	2º	Normal

FS was calculated using the following expression:

$$FS = \frac{cL+[W\cos\theta+F\cos(90-\theta-\delta)]\tan\phi+F\sin(90-\theta-\delta)}{W\sin\theta} \quad \textbf{(16)}$$

Where the terms are defined in Figure 4 and Table 3 and where L is the length of the joint.

In this case, the target function is defined by Equation 16. It is evaluated 2N+1 times, in order to calculate the derivatives defined in Equation 15, and the results are presented in Table 4.

Table 4: FOSM Results

FS	Dens	Friction	Cohesion	Joint dip	Bolt Force	Bolt inc	df/dxi	(df/dxi)^2 Var[xi]
1.328	27.0	30.0	9.6	35.0	70.0	10.0	-	-
1.282	29.7	30.0	9.6	35.0	70.0	10.0	-0.0188	0.005167
1.384	24.3	30.0	9.6	35.0	70.0	10.0	-	-
1.460	27.0	33.6	9.6	35.0	70.0	10.0	0.03541	0.032504
1.205	27.0	26.4	9.6	35.0	70.0	10.0	-	-
1.472	27.0	30.0	13.4	35.0	70.0	10.0	0.03785	0.041913
1.183	27.0	30.0	5.7	35.0	70.0	10.0	-	-
1.390	27.0	30.0	9.6	38.5	70.0	10.0	0.00389	0.000371
1.363	27.0	30.0	9.6	31.5	70.0	10.0	-	-
1.342	27.0	30.0	9.6	35.0	77.0	10.0	0.00201	0.000399
1.314	27.0	30.0	9.6	35.0	63.0	10.0	-	-
1.326	27.0	30.0	9.6	35.0	70.0	12.0	-0.0007	3.49E-06
1.329	27.0	30.0	9.6	35.0	70.0	8.0		-
							Var[FS]	**0.0804**

The expected value of FS is E[FS]=1.328, and is calculated using Equation 13 and corresponds to the evaluation of Equation 16 using the average variables, if covariance is not considered. The variance of FS is Var[FS]=0.0804 and is calculated using Equations 14 and 15.

Methods like FOSM and Point Estimate Methods (described below) provide a way to assess the average and variance of FS, but they do not provide additional information about the actual distribution of the main variable, in this case FS.

To assess Pf an assumption must be made about the distribution of FS. In this example, both a normal, and log normal distribution have been assumed and the results are shown in Figure 5. For this problem, failure is defined as a configuration of geometry, density and rock bolt properties with FS<1.0, therefore P_f is calculated as the area below the curve for FS<1.0.

For both distributions P_f is: Normal Distribution: P_f=0.123 Log Normal Distribution: P_f=0.107

As is expected the P_f is a function of the distribution assumed.

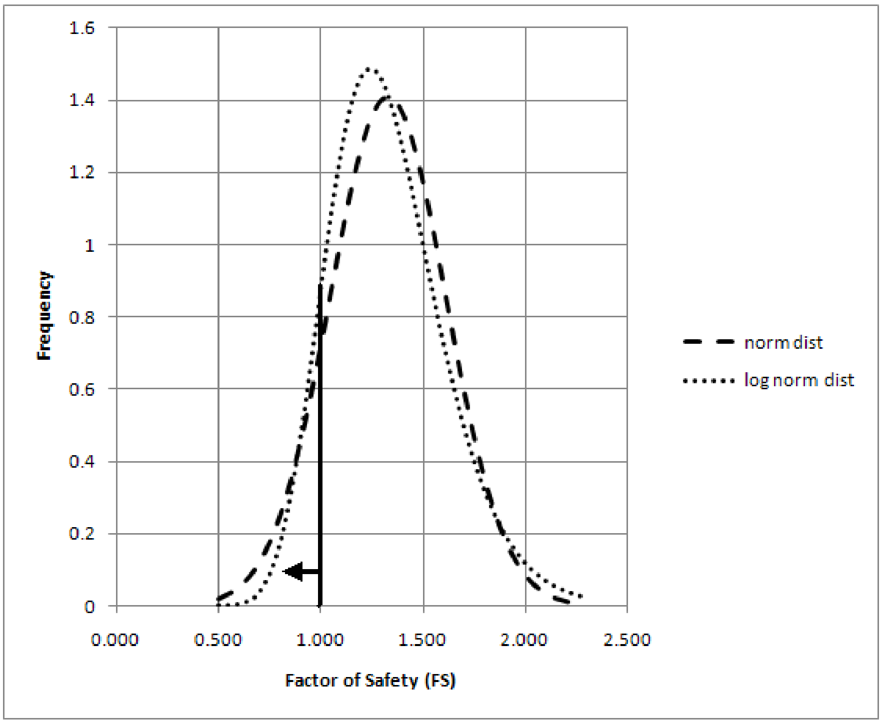

Figure 5: Normal and Log Normal Distribution for Factor of Safety

2.1.3 Point Estimate Method (PEM)

The Point Estimate Method developed by Rosenblueth (1975), Harr (1987) is used to assess the expected value E[y] and the variance Var[y], where y is a function of N variables (Equation 17).

$$y = f(x_1, x_2, x_3, \dots x_N) \tag{17}$$

For each variable x_i there are two evaluation points They are defined as $x_{i+} = \bar{x}_i + \sigma_{xi}$ and $x_{i-} = \bar{x}_i - \sigma_{xi}$. The function y is evaluated for all the possible combinations of the evaluation points, this requires 2^N evaluations. For instance, for N=2, y is evaluated in the following points (Equation 18):

$$\begin{gathered}(\bar{x}_1 + \sigma_{x1}, \bar{x}_2 + \sigma_{x2}) \\ (\bar{x}_1 - \sigma_{x1}, \bar{x}_2 + \sigma_{x2}) \\ (\bar{x}_1 + \sigma_{x1}, \bar{x}_2 - \sigma_{x2}) \\ (\bar{x}_1 - \sigma_{x1}, \bar{x}_2 - \sigma_{x2})\end{gathered} \tag{18}$$

The expected value for y and its variance are calculated using Equations 19 and 20

$$E[y] \approx \sum_{k=1}^{2^N} p_k y_k \tag{19}$$

$$Var[y] \approx \sum_{k=1}^{2^N} p_k\, y_k^2 - E[y]^2 \tag{20}$$

and, where p_k is calculated by Equation 21:

$$p_k = \frac{1}{2^N}\left[1 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} S_i S_j \rho_{ij}\right] \tag{21}$$

Where ρ_{ij} is the correlation coefficient between x_i and x_j, S_i=+1 for points greater than the mean and S_i=-1 for points smaller than the mean. For instance for the third equation of equation 18, S_1=+1 and S_2=-1.

Similar to the FOSM method, PEM provides an estimate for the average and the Variation of the main variable. There is no additional information about the actual distribution of the variable. To assess the probability of failure an assumption about the distribution of y must be made.

PEM was applied to the problem presented in Figure 4. The results are summarised in Table 5.

2.1.4 Modified Point Estimate Method (mPEM)

Modified Point Estimate Method is a variation of the PEM that reduce the number of analyses required. PEM requires 2^N analyses to assess the probability of failure, mPEM requires only 2N. The method was presented by Harr (1989) using an example. The equations used in the method are presented here.

$$y = f(x_1, x_2, x_3, \ldots x_N) \tag{22}$$

Equation 22 represents the relationship between the input variables xi and output parameters y. The parameter y could be FS and x_i could be density, cohesion, friction or other variables in the model. The function f() represents a particular analysis to calculate y, it could be limit equilibrium analysis, finite element analysis, or an analytical expression.

For each variable the average (equation 23) and standard deviation are known (Equation 24)

$$\bar{X} = (\overline{x_1}, \overline{x_2}, \overline{x_3}, \ldots \overline{x_N}) \tag{23}$$

$$S = (\sigma_1, \sigma_2, \sigma_3, \ldots \sigma_N) \tag{24}$$

The correlation between the variables is also known. K represents the correlation matrix, where the term k_{ij} is the correlation between the variables i and j.

The method requires solving the Eigen problem indicated by Equation 25, where the subindex i represents an eigenvalue λ and eigenvector ϕ related to variable i.

$$(K - \lambda_i I)\phi_i = 0 \tag{25}$$

The eigenvalues λ_i are normalised, such $\Sigma\lambda_i = N$.

For each eigenvalue $\lambda_{i,}$ and eigenvector $\phi_{i,}$ the target function y is evaluated for the evaluation points defined by Equations 26 and 27:

$$y_i^+ = f(\bar{x} + \phi_i \sqrt{N} \sigma_i) \tag{26}$$

$$y_i^- = f(\bar{x} - \phi_i \sqrt{N} \sigma_i) \tag{27}$$

Where y is the result parameter calculated for a specific analysis i.e. FS of slope stability, beam deflection, or other. The expected values for y and y^2 are calculated using Equations 28 and 29

$$E[y_i, \lambda_i] = \frac{y_i^+ + y_i^-}{2} \frac{\lambda_i}{N} \tag{28}$$

$$E[y_i^2, \lambda_i] = \frac{(y_i^+)^2 + (y_i^-)^2}{2} \frac{\lambda_i}{N} \tag{29}$$

The expected value of the variable y and its Variance can be calculated using Equations 30 and 31.

$$E[y] = \textstyle\sum_1^N E[y_i, \lambda_i] \tag{30}$$

$$Var[y] = \textstyle\sum_1^N E[y_i^2, \lambda_i] - (\sum_1^N [y_i, \lambda_i])^2 \tag{31}$$

When the expected value E[y] and variance Var[y] are calculated it is possible to assess the probability of failure assuming a distribution for the variable y (normal, log normal or other).

mPEM was applied to the problem shown in Figure 4, the results are presented in Table 5 and discussed in the next section.

2.2 DISCUSSION

Four methods were used to analyse the same problem, the results of which are summarised in Table 5. The case of friction and cohesion correlated (ρ=-0.5) was included. Two Monte Carlo simulations (MC) with 20000 trials are also included, in the first one (MC1) the density, friction and cohesion have a log normal distribution; in the second simulation (MC2) all the variables have normal distribution.

The three alternative methods are compared with the results obtained for MC simulation. Table 5 shows the variability in results obtained from the different methods when applied to the same problem.

FOSM tend to overestimate the probability of failure even though the assessment for the variance was close to the values calculated with MC. FOSM assumes that the average value of the target function is the same as the value obtained evaluating average values for the input parameters. This is not always true.

PEM overestimates P_f when compared to MC results. It produces a result closer to MC when all the parameters have a normal distribution.

mPEM falls in between FOSM and PEM, producing better results than FOSM with almost the same number of analyses, but tends to overestimate P_f like the other two methods.

Table 5: Summary of Results for Different Methods

Correlation	Parameter	FOSM	PEM	mPEM	MC1 (n=20000)	MC2 (n=20000)
ρ=0	E[FS]	1.328	1.386	1.420	1.396	1.389
	σ [FS]	0.283	0.216	0.297	0.270	0.250
	P_f (normal)	0.123	0.037	0.079	0.0102	0.0272
	P_f (log normal)	0.107	0.021	0.056		
ρ=-0.5	E[FS]	1.328	1.386	1.420	1.389	1.395
	σ [FS]	0.249	0.163	0.263	0.221	0.231
	P_f (normal)	0.094	0.009	0.055	0.0008	0.0066
	P_f (log normal)	0.075	0.003	0.034		

When MC simulations are compared, is interesting to note the influence that the correlation and distribution assumed for the input parameters has in the final result. Changing the correlation from 0 to -0.5 reduces the P_f 12 times.

2.3 INTERPRETATION OF THE PROBABILITY OF FAILURE

The probability of failure (expressed in %) can be interpreted as the number of failure we should experience on average in 100 "trials". This is difficult to relate to overall pit slope stability, in a typical open pit, one "trial" of a combination of height, rock type, jointing and water is present in the pit. At inter-ramp scale the number of "trials" can be considered higher but still will be only a few.

At bench scale, the situation is different. If the probabilistic analysis was carried out for a particular rock type at bench scale; it is reasonable to think that there are many "trials" of the design along a wall with similar rock type and joint conditions. For instance, if we assume the width of the failure at bench scale is similar to the height of the bench, in a 150m long wall with 6 benches 10 m high there are 90 "trials" for the design.

If the probability to have a failure in one bench is equal to p, then the probability to have n or more failures can be calculated with Equation (32).

$$P(X \geq n) = 1 - \sum_{i=0}^{N} \frac{N!}{i!(N-i)!} p^i (1-p)^{N-i} \quad \textbf{(32)}$$

Where: N is the total number of "trials".

If Equation 32 is applied with N=90 and p=40% the results are presented in Figure 6.

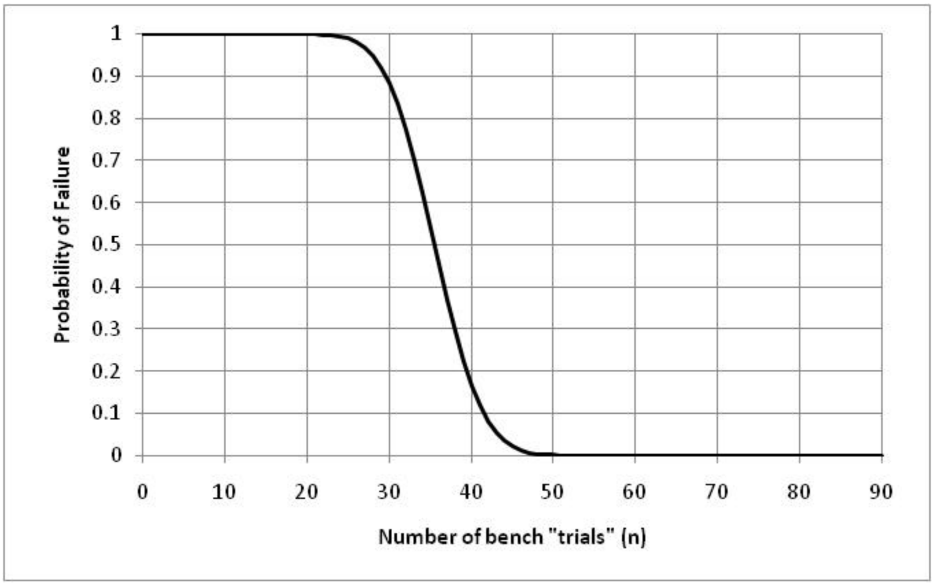

Figure 6: Probability of n Failures and Bench Scale.

From previous graphs is possible to conclude that is almost certain that the wall will experience 25 or fewer bench failures, there is a 50% probability the number of failures is less than 36 and it is unlikely that there will be more than 45 bench failures. Despite the high probability of failure for a bench (40%) it is unlikely to expect that 50% or more of the length of the benches will fail.

3 ACCEPTABLE PROBABILITY OF FAILURE IN SLOPE DESIGN

Steffen (2006) suggested that the pit design criteria should be based on risk rather than factor of safety or probability of failure. The risk is calculated considering the probability of failure and the consequence of the failure.

Consequence should include fatality risk and economical losses. In this paper, we have concentrated on the probability of failure as design criteria in open pits, no comments are included about risk values to be used as design criteria.

In open pits is possible to identify three different slopes scales.

- Bench scale
- Inter ramp slope
- Overall slope

Without quantifying the risk of failure for each case, it is possible, at least intuitively, recognise that a different probability of failure should be used for each case because the impact (consequence) of failure for each case is different.

Sjoberg (1999) presents Table 6, were the consequence of the failure is related to probability of failure and Reliability Index β.

$$\beta = \frac{E[FS]-1}{\sigma[FS]} \quad \textbf{(33)}$$

Table 6: Acceptance Criteria for Rock Slopes (after Sjoberg, 1999)

Category and Consequence of Failure	**Example**	**Reliability Index β**	**Probability of Failure P_f**
Not serious	Non-critical benches	1.4	0.1
Moderately serious	Semi-permanent slopes	2.3	0.01 to 0.02
Very serious	High/permanent slopes	3.2	0.003

Schellman (2006) presents probabilities of failure based on the volume of material involved in the failure. Table 7 shows the probabilities used in Mantoverde Mine.

Table 7: Acceptance Criteria for Rock Slopes at Mantoverde Mine

Mass Involved in Failure (t/m)	**Factor of Safety [FS]**	**Probability of Failure [P_f]**
<15,000	>1.20	<0.12
15,000 – 30,000	>1.25	<0.10
>30,000	>1.30	<0.08

Pothitos (2007) presents Table 8 indicating the probability of failure used in Ok Tedi mine.

Table 8: Design Probability of Failure for Mine Slope Design – Ok Tedi.

Design Situation			**Probability of Failure Commonly Used or Accepted in Practice**	
Design Element	**Applicability**	**Geotechnical Conditions**	**Range (%)**	**Preferred Value**
Bench Slope	General	-	10 to 50	-
	-	Continuous Defects	0 to 10	10
	-	Discontinuous Defects	10 to 50	20 – 30
Overall or inter-ramp slope	General	-	1 to 3	-
Overall or inter-ramp including haul road or key infrastructure	-	-	<1	-

Tables 6 to 8 indirectly include failure size in the probability of failure accepted for design. When the failure size increases or a large portion of the pit is involved in a potential failure, the probability of failure is reduced.

Kirsten (1983) shows a different table with probability of failure related to serviceable life, public liability and monitoring (Table 9).

Table 9: Comparative Significance of Probability of Failure.

Probability of Failure (%)	Design Criteria on Basis of which Probability of Failure is Established			Aspects of Natural Situation in Terms of which Probability of Failure can be Assessed	
	Serviceable Life	Public Liability	Minimum Surveillance Required	Frequency of Evident Slope Failures	Rate[1] of Evidently Unstable Movements
50 to 100	Effectively zero	Public access forbidden	Serves no purpose (excessive probability tantamount to failure	Slope failures generally evident	Abundant evidence of creeping valley sides
20 to 50	Very short term (temporary open pit mines[2] - untenable risk of failure in temporary civil	Public access forcibly prevented	Continuous monitoring with intensive sophisticated instruments	Significant number of unstable slopes works	Clear evidence of creeping valley sides
10 to 20	Very short term (quasi-temporary slopes in open pit mines - undesirable risk of failure in quasi-temporary civil works)	Public access actively prevented	Continuous monitoring with sophisticated instruments	Some unstable slopes evident	Some evidence of slowly creeping valley sides
5 to 10	Short term (semi-temporary slopes in open pit mines[3], quarries or civil works)	Public access prevented	Conscious superficial monitoring	No ready evidence of unstable slopes	Extremely slowly creeping valley sides not readily evident
1.5 to 5	Medium term (semi-permanent slopes)	Public access allowed	Incidental superficial monitoring	No unstable slopes evident	No unstable movements evident
Less than 0.5	Very long term (permanent slopes)	Public access free	No monitoring required	Stable slopes	No movements

Note:

1 The rate of movement implied in the natural situation is with regard to geological time. The quantitative assessments are further given with regard to a significant number of locations at which failure can occur.

2 The lateral extent of a location where failure can occur is of the order of the height of the affected slope.

3 Small open pit mines lie in the range of 5% to 15% allowable probability of failure depending upon the extent of monitoring as given. The corresponding range for large open pit mines is 15% to 30%.

The previous tables, in general, do not have a common definition for the slopes and recommend different probability of failure for similar conditions. Application of the previous tables to an open pit produces a common range of probability of failure for use in design as presented in Table 10. It is important to keep in mind that Schellman and Pothitos presented probabilities of failure used for a specific mine; Sjoberg and Kisten present general recommendations of probability of failure for slope design.

Table 10: Summary of Probability of Failure in Percentage

Design Element	Sjoberg	Schellman	Pothitos	Kirsten	Recommended
Bench	10	12	10 – 50	20 to 50	15 to 30
Inter-ramp	1 to 2	8 to 10	1 to 3 <1*	5 to 10	2 to 5 <1*
Overall Slope	0.3	<8	1 to 3 <1*	1.5 to 5	1 to 2 <1*

* Overall or inter-ramp including haul road or key infrastructure

4 CONCLUSIONS

This paper presents the equations required to apply the mPEM developed by Harr (1989). The method was compared with others: FOSM, PEM and MC.

MC is used with a large number of simulations; this allows us to assume that the results obtained are close to the exact solution. It is important to note the MC simulations do not require an assumption about the distribution of the target function (FS or other). If a large enough number of runs is done, the true distribution of the target function is modelled with enough precision. MC is not suitable for large analysis where one run requires hours or in some cases days for computer time. Simplified approach has to be made in those cases.

mPEM requires 2n evaluation of the target function (analyses), very similar to the 2n+1 analyses required with FOSM but mPEM produces a better results. PEM requires 2^n analyses producing results closer to the values calculated with MC.

Probabilistic analysis appears more appealing than deterministic analysis, but introduces another layer of uncertainties that should be considered when probabilistic methods are used. Based on the results obtained in the example shown in this paper, the following three factors have an influence in the final result:

- Correlation. The correlation between different input parameters should be addressed. For the examples shown, P_f changes with the correlation assumed between cohesion and friction.
- Distribution function of the Factor of Safety or target function used in design. Some of the probabilistic methods only provide an assessment for the average and variance of the FS. To calculate the probability of failure, a distribution function must be assumed (normal, log-normal or other). The choice of the distribution function changes the resultant P_f.
- Method used to assess probability of failure. It has been shown in the example analyses that there is variability in the results depending on the method used. This variability could be large enough that in some cases we might reject a design, but using a different method we may find the design satisfactory.

The paper includes an examination (non exhaustive) of Probability of Failure used in slope design, a range of values is recommended to be used in design of bench, inter-ramp and overall slope angle.

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